

Hypothesis Testing

February 25, 2020

Data Science CSCI 1951A

Brown University

Instructor: Ellie Pavlick

HTAs: Josh Levin, Diane Mutako, Sol Zitter

Announcements

-?

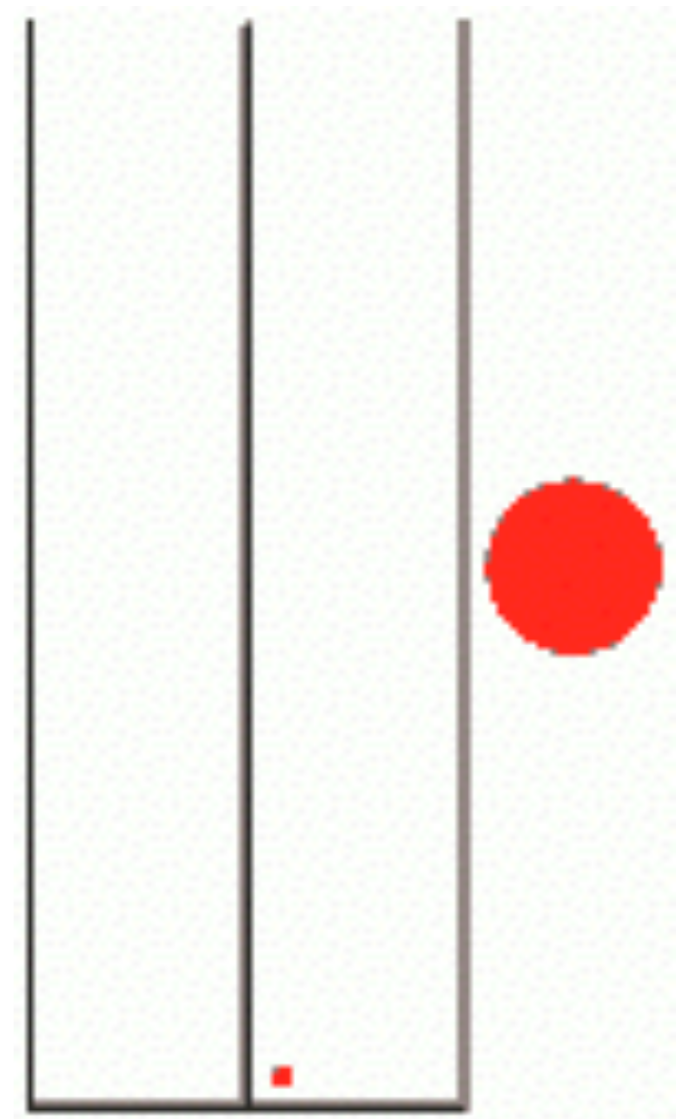
Today

- **Two quick preliminaries: LoLN and CLT**
- Follow up from last time
- Common tests: chi-squared test, z-test/t-tests

Law of Large Numbers

- If you perform the same experiment a large number of times, the *average* will converge to the expected value
- Assumes that errors are “random” and uncorrelated, so will balance out over time

$$\bar{X}_n = \frac{1}{n} (X_1 + \dots + X_n)$$
$$\bar{X}_n \rightarrow \mu \text{ as } n \rightarrow \infty$$

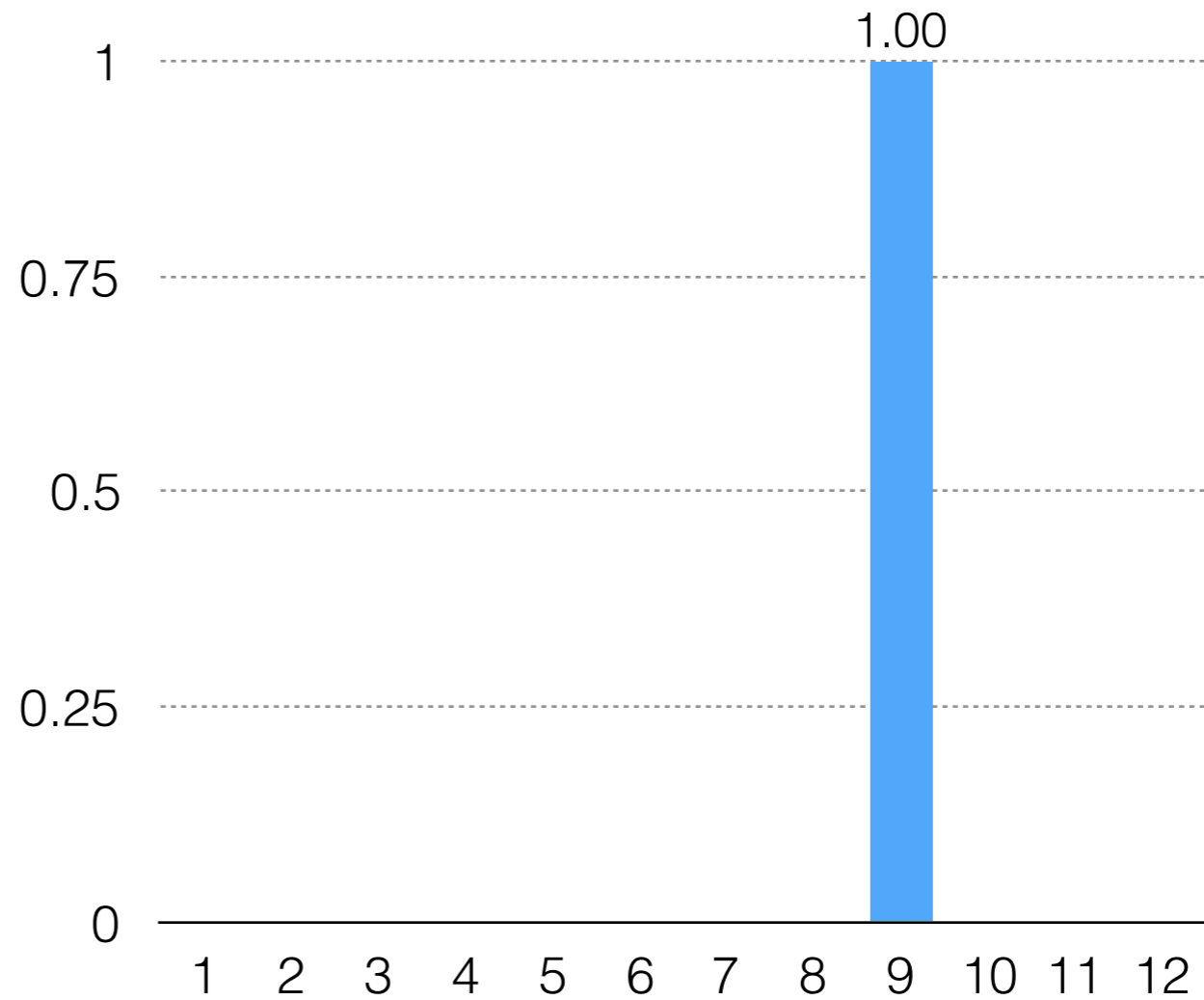


Central Limit Theorem

- Given $X_1 \dots X_n$
- Not only does a $\bar{X}_n \rightarrow \mu$ as $n \rightarrow \infty$
- But the distribution approaches a normal distribution

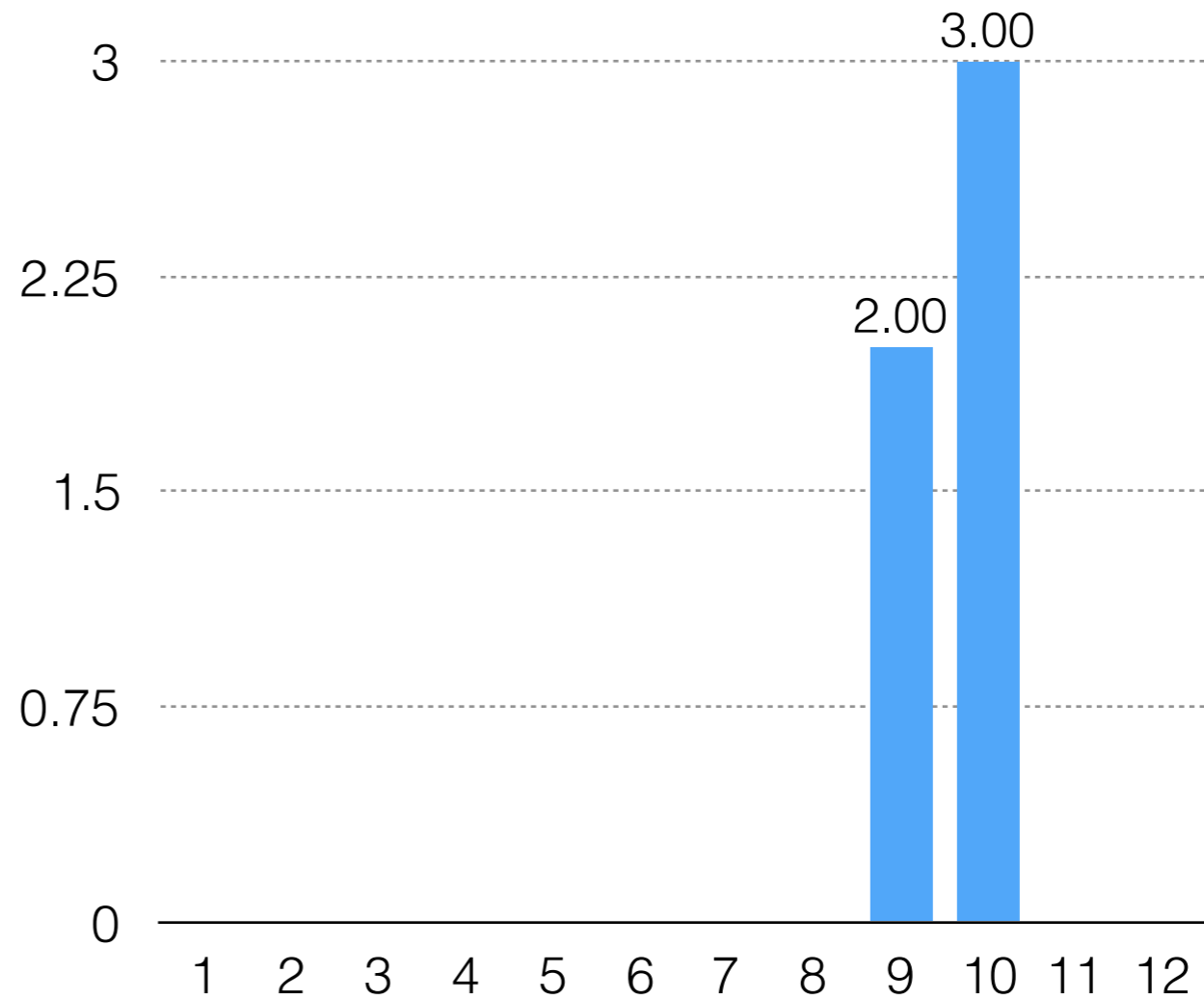
Central Limit Theorem

c **d** **b** **d**
b **a** **d** **c**
b **c** **b** **d**



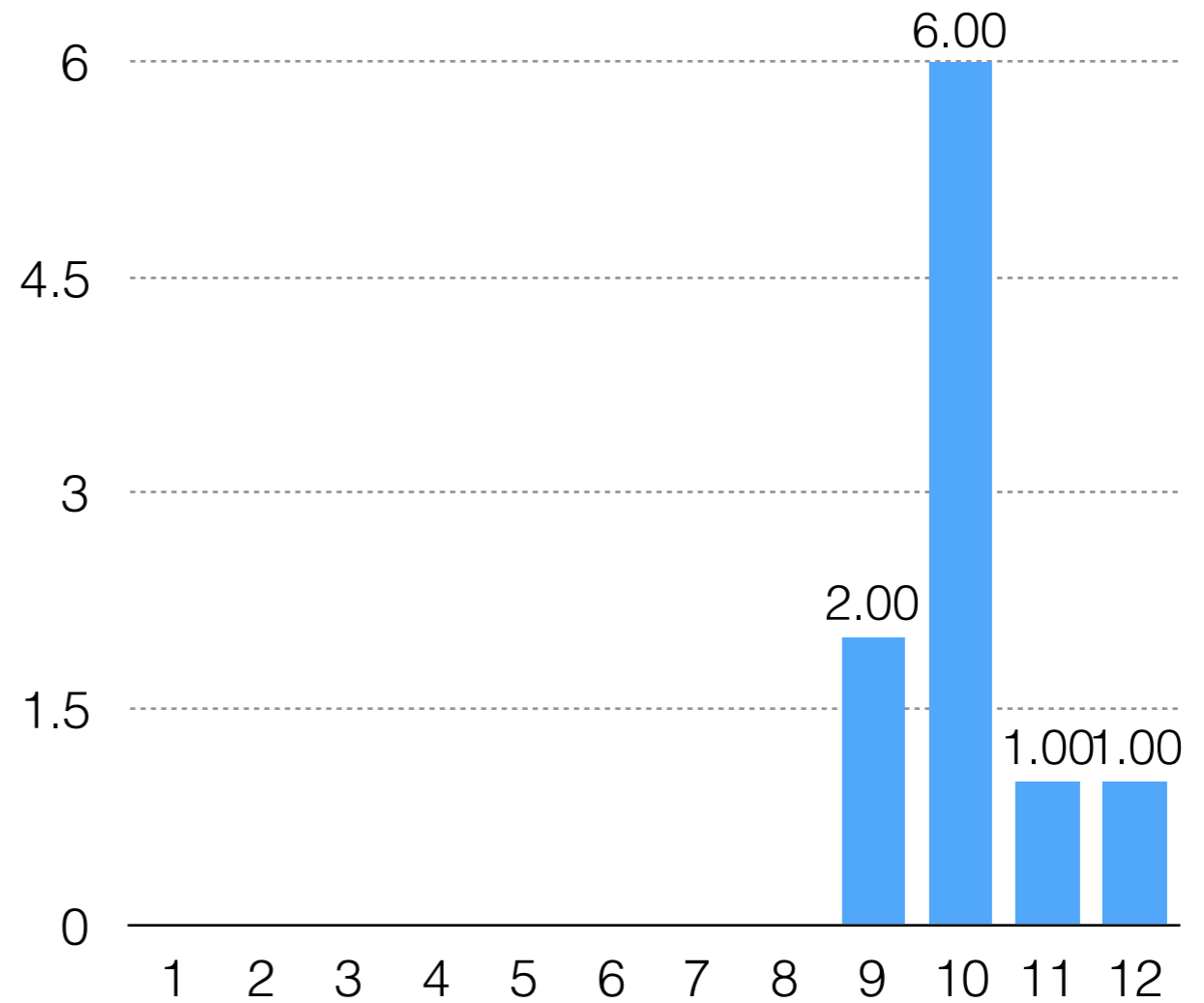
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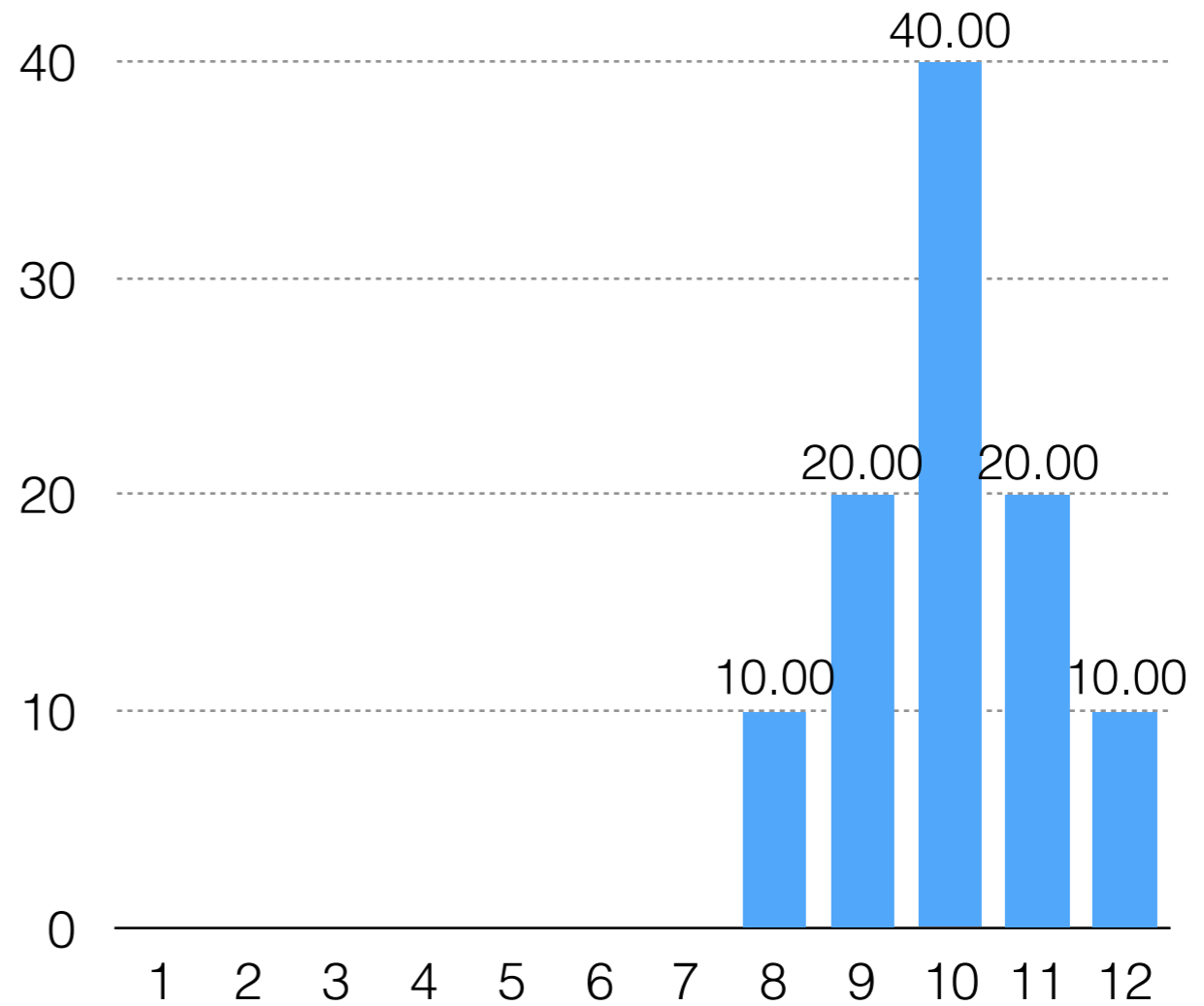
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Central Limit Theorem

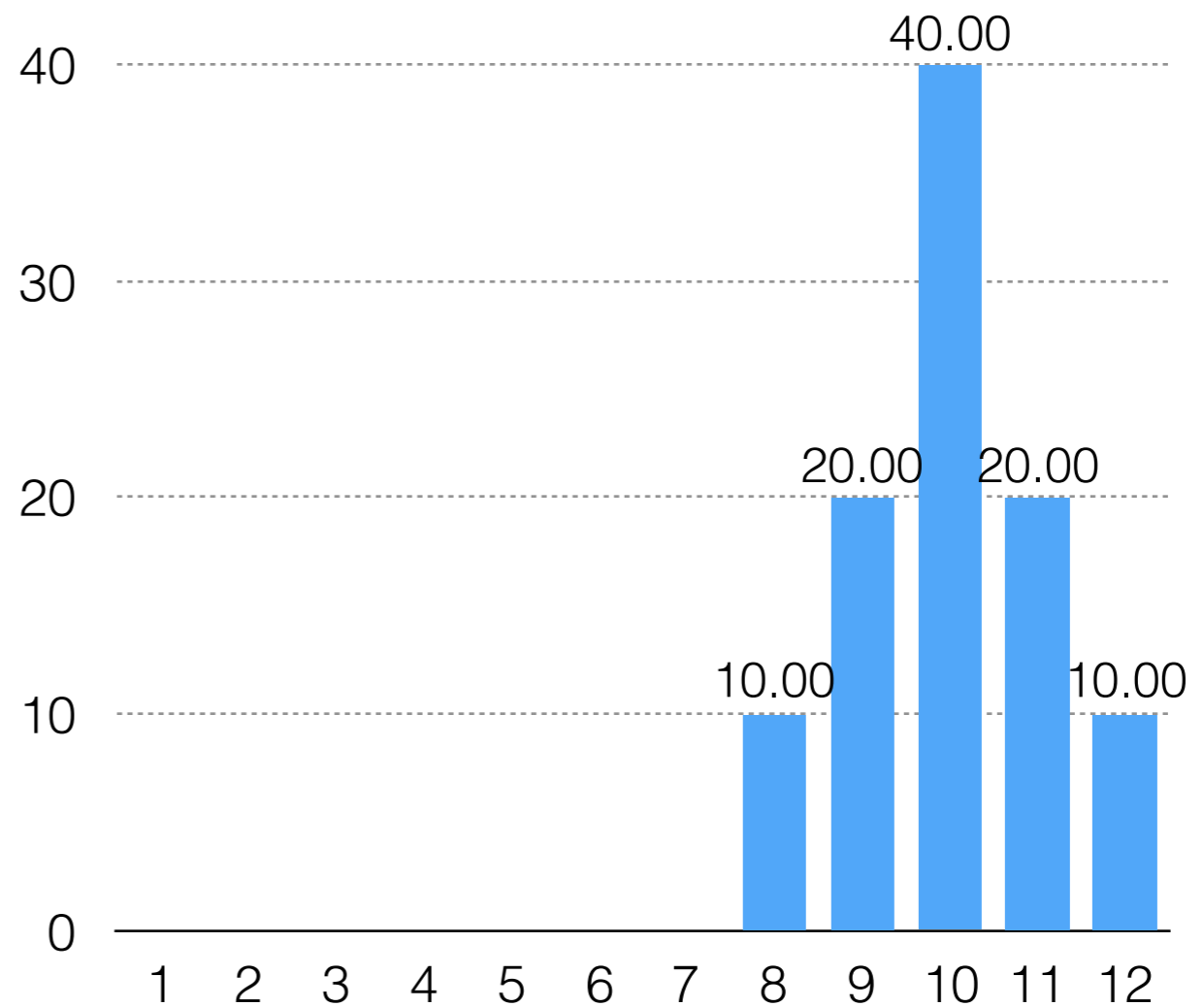
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Central Limit Theorem

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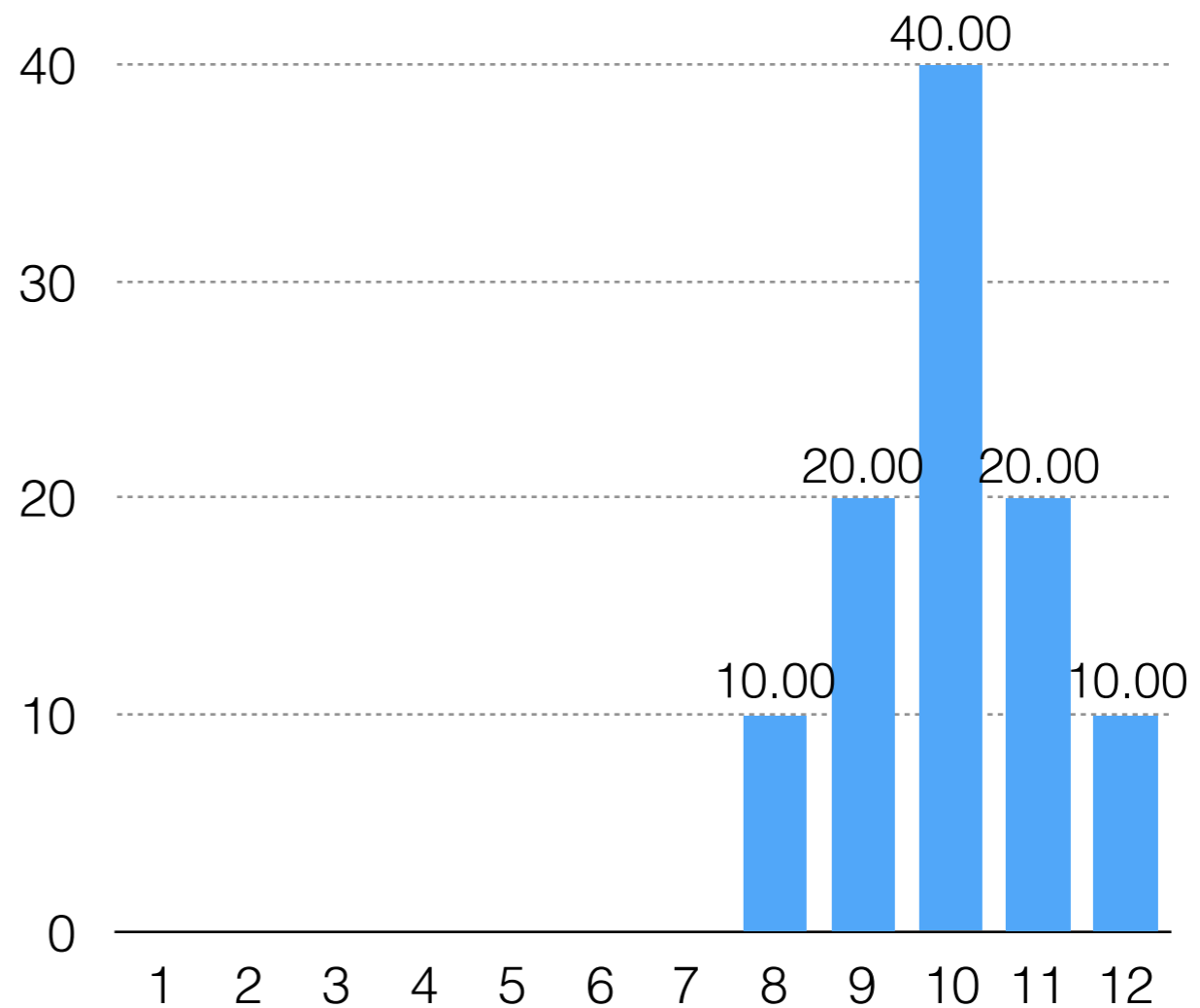
I.e. test statistics are often normally distributed...



Central Limit Theorem

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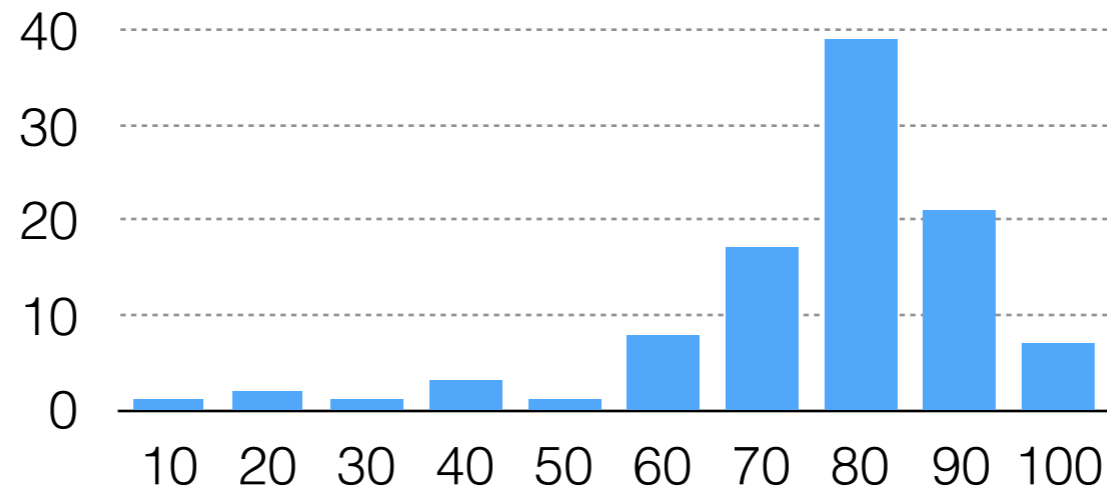
Can apply statistical methods designed for normal distributions even when underlying distribution is not normal



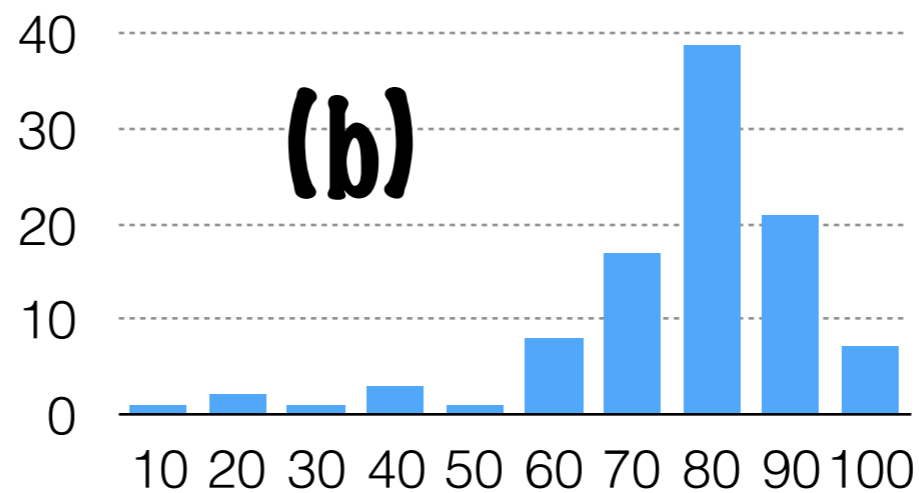
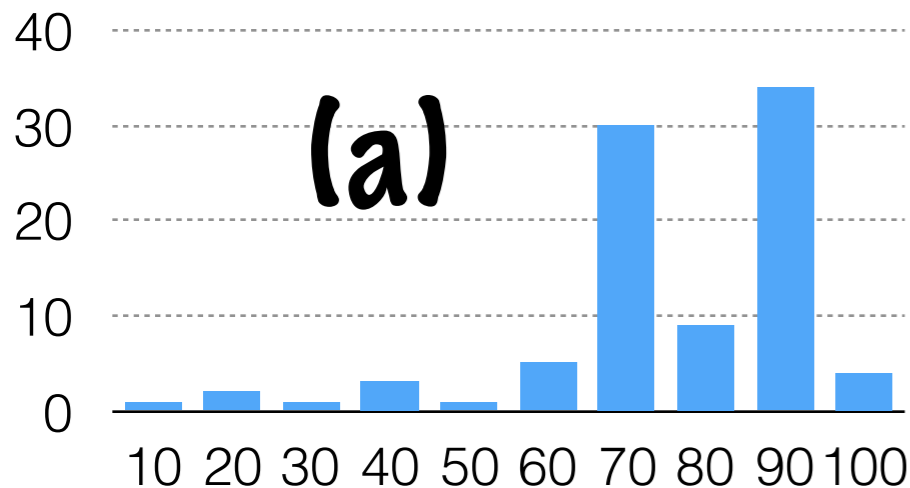
Clicker Question!

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Every year, I compute the mean grade in my class. I never change the material or my methods for evaluating because, lazy. Over the 439 years that I have been teaching this class, this has resulted in the below distribution.



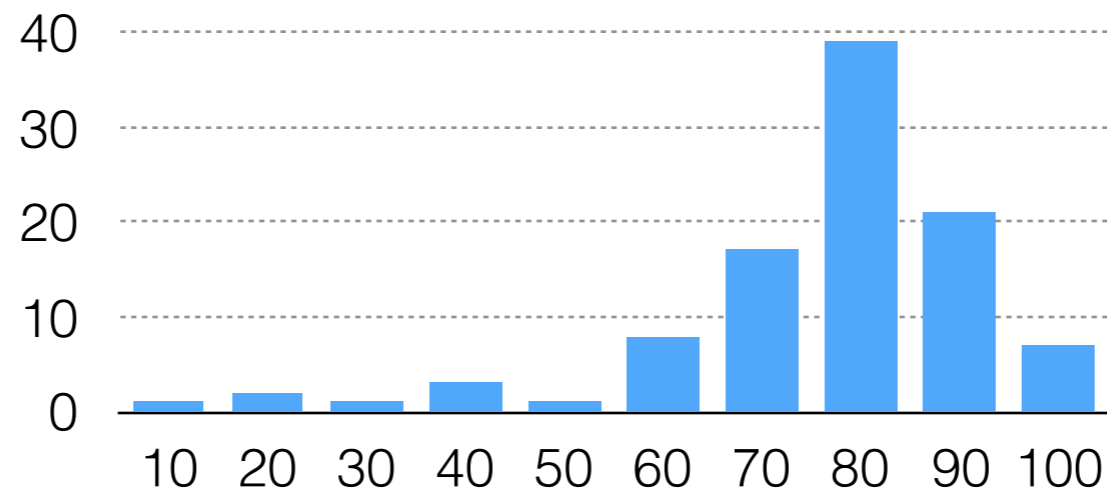
Which of these is mostly like the typical distribution on any given year?



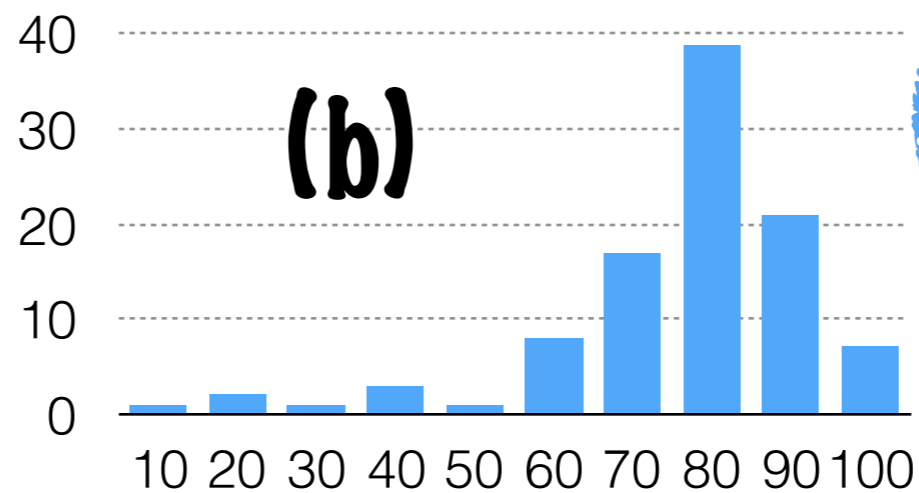
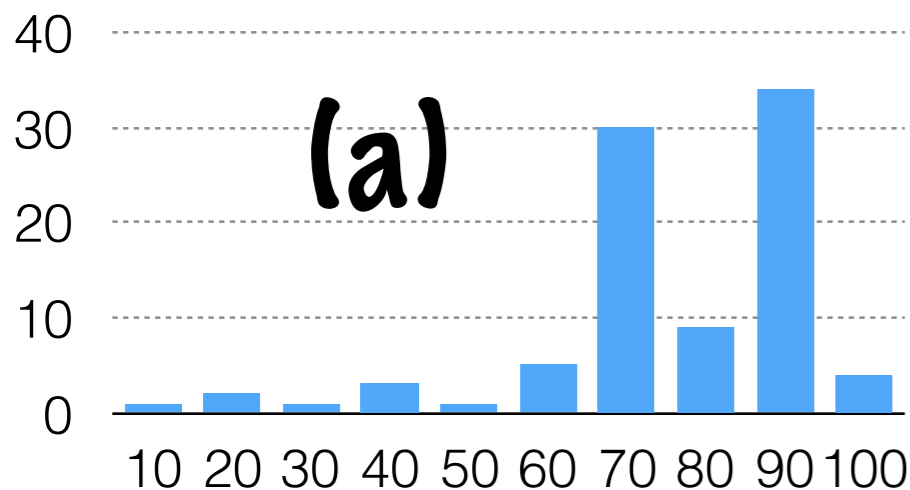
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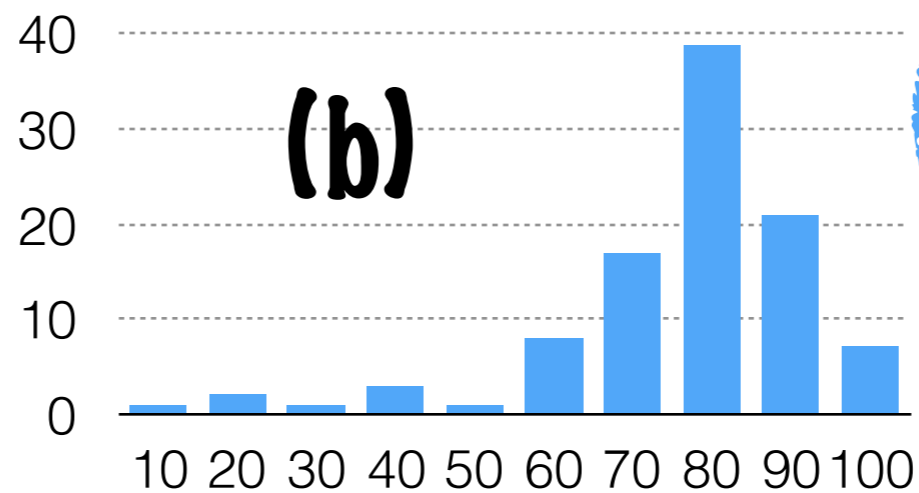
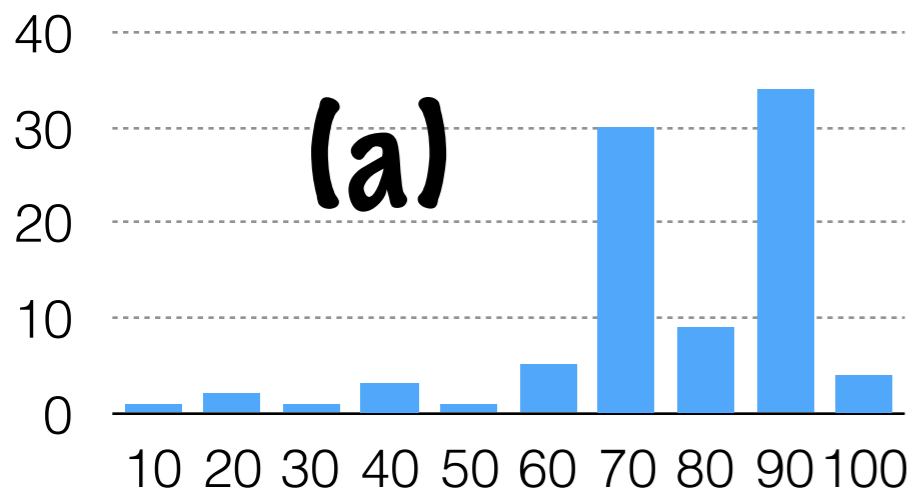
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Central Limit Theorem: repeated measures of mean will be normally distributed, doesn't assume the population over which you are taking the mean is normally distributed.

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Hypothesis Testing in General

Thing you can model
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Matters for how you compute
p-values...more soon

Hypothesis Testing in General

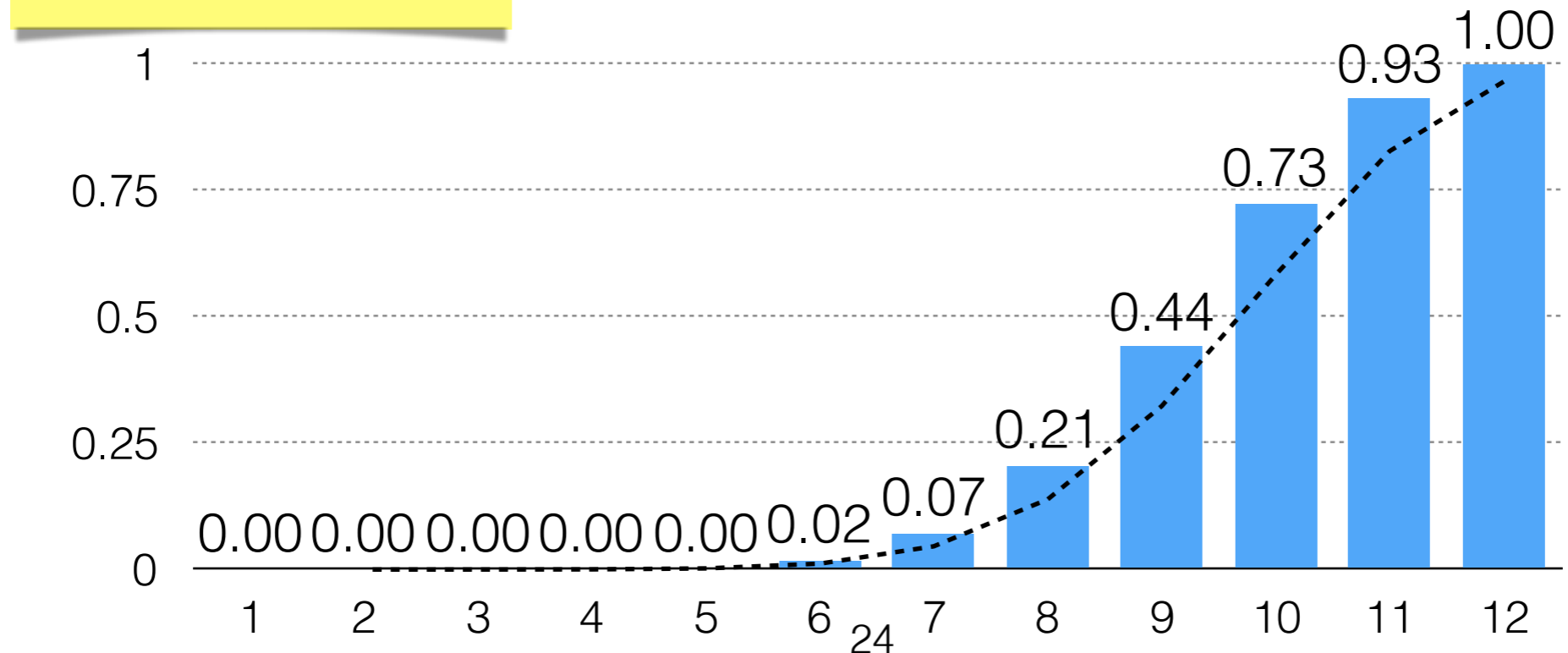
- Assume the null hypothesis is true—i.e., don't deviate from status quo without good reason :)
- If there is enough evidence to suggest that H_0 is highly unlikely, then we can say we “reject the null hypothesis”
- If there is not enough evidence, we “fail to reject it”
- We don't “accept” or “prove” H_0 or H_a

Hypothesis Testing in General

- Determine an appropriate test statistic, given your null hypothesis
- Come up with a theoretical distribution of that test statistic (often, this work has been done for you)
- Compute the likelihood of your test statistic, given that theoretical distribution

Hypothesis Testing

"I swear literally like 80% of the answers are just (b)"



Hypothesis Testing

Hypothesis

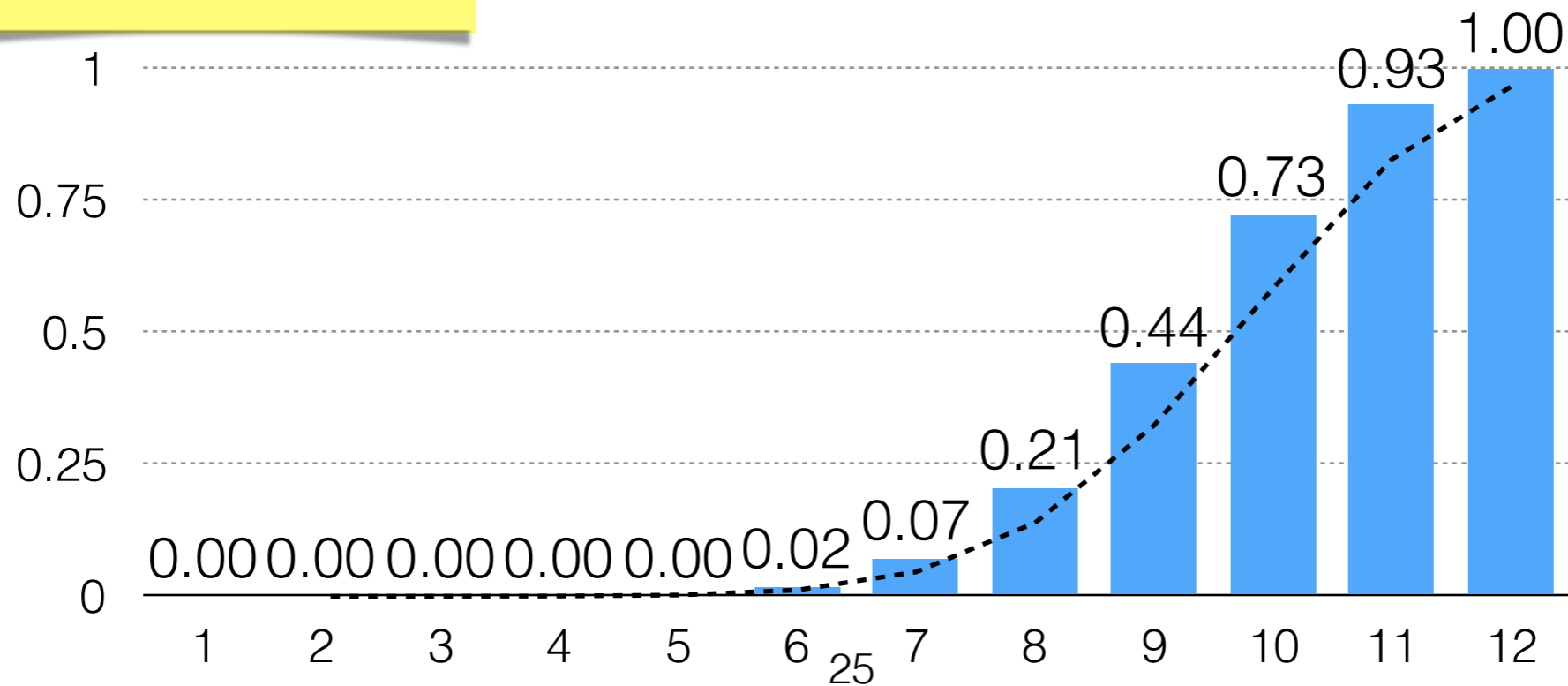
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→ 4
cdf

$\langle \Omega, F, P \rangle$

{b, not b} $p(B) = 0.8$



Hypothesis Testing

Observation/Sample

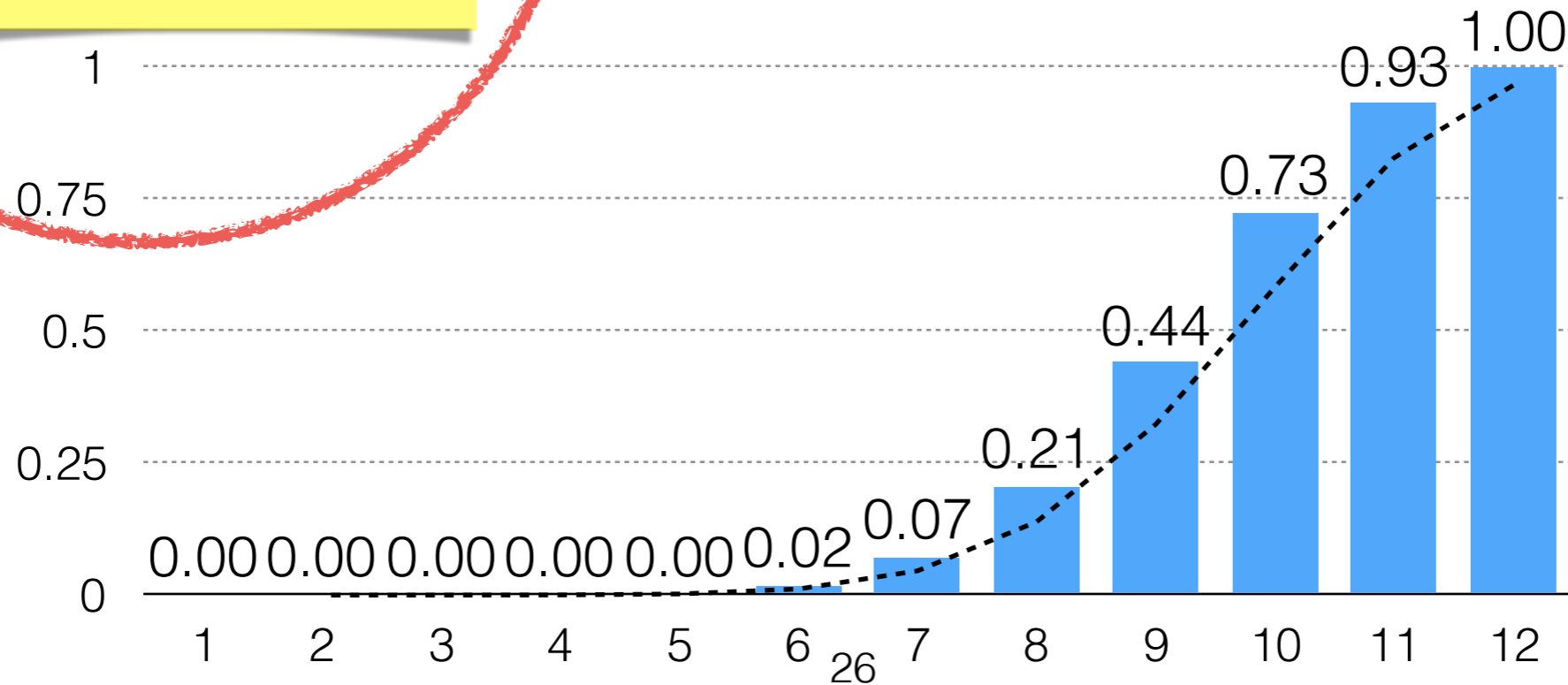
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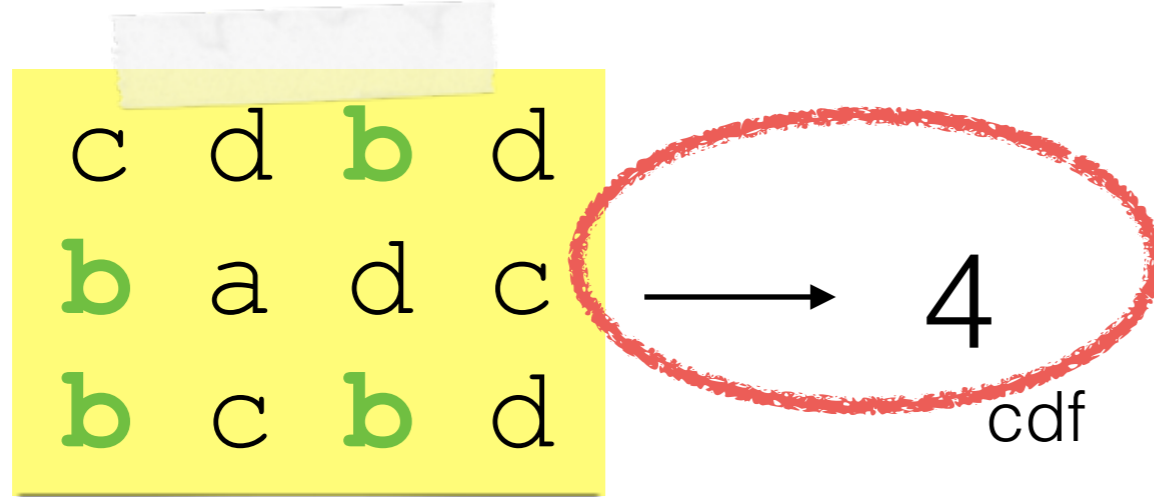
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Hypothesis Testing

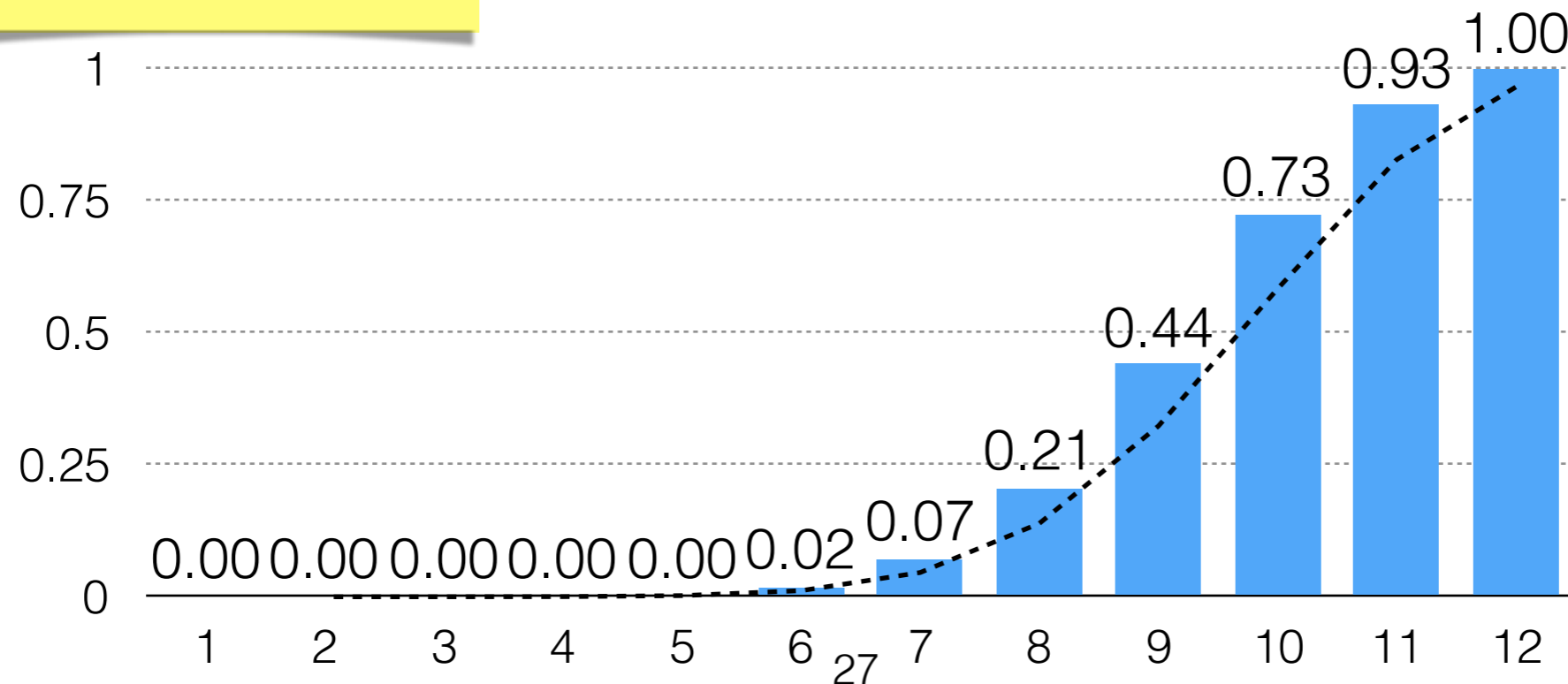
Test Statistic

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Hypothesis Testing

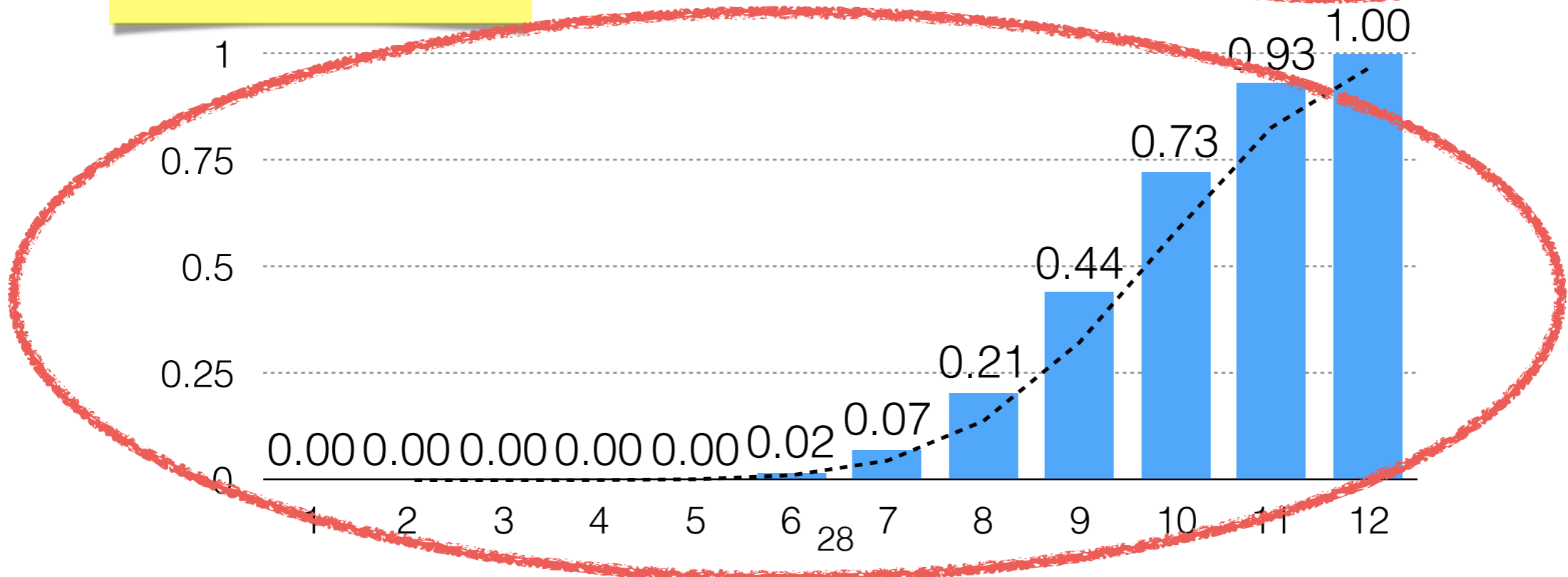
Theoretical Distribution

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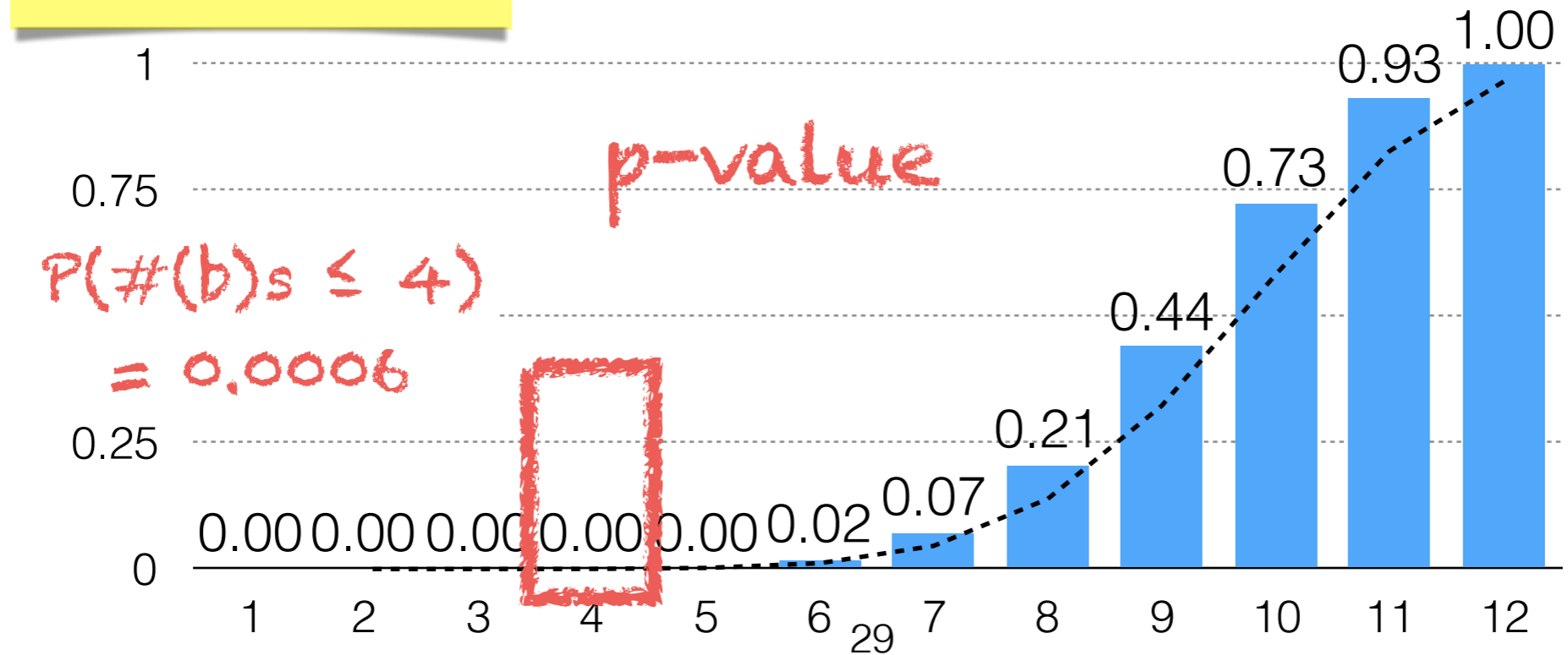
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 - e.g. are men taller than women, are blue states more populated than red states, do CS work harder than other majors (::rolling_eyes::)

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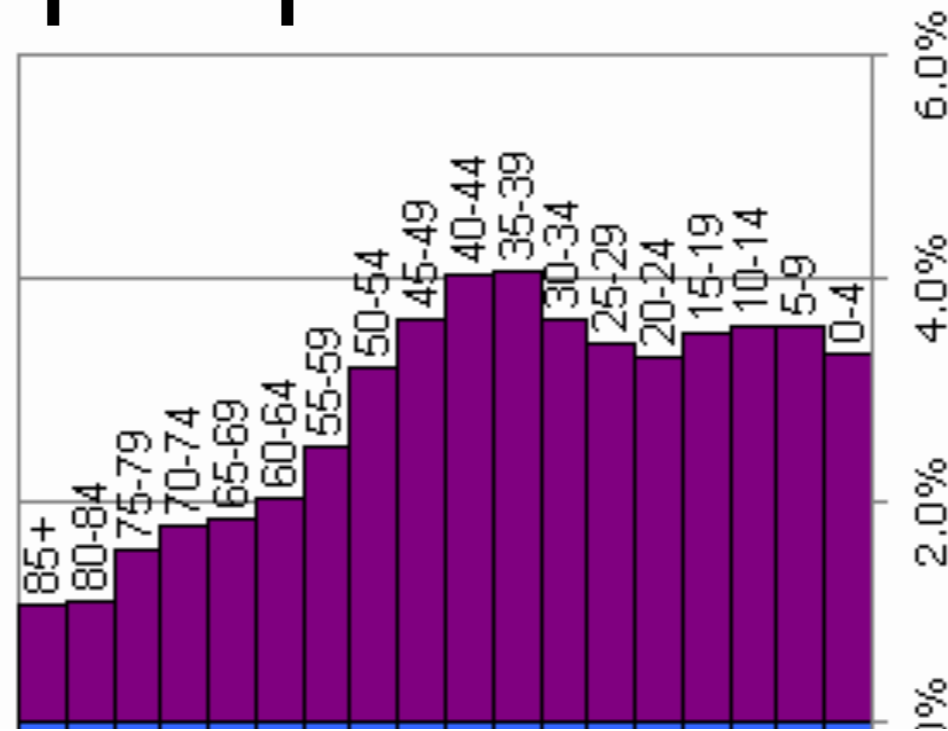
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Test for population means

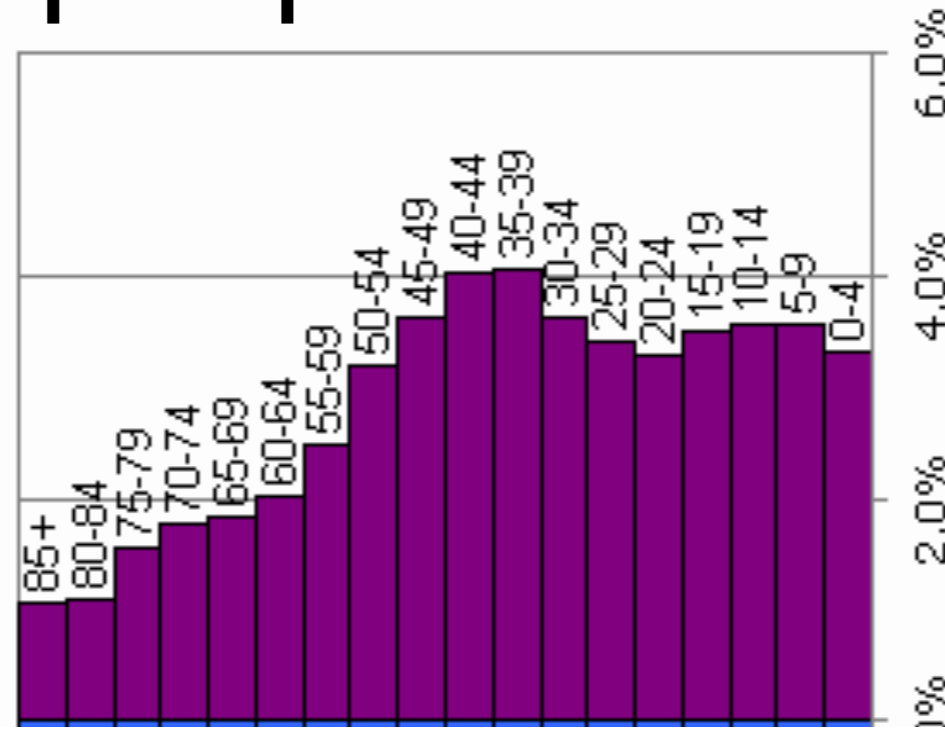
Test for population means



Distribution of ages in the US

Hypothesis: Mean age is 35.

Test for population means



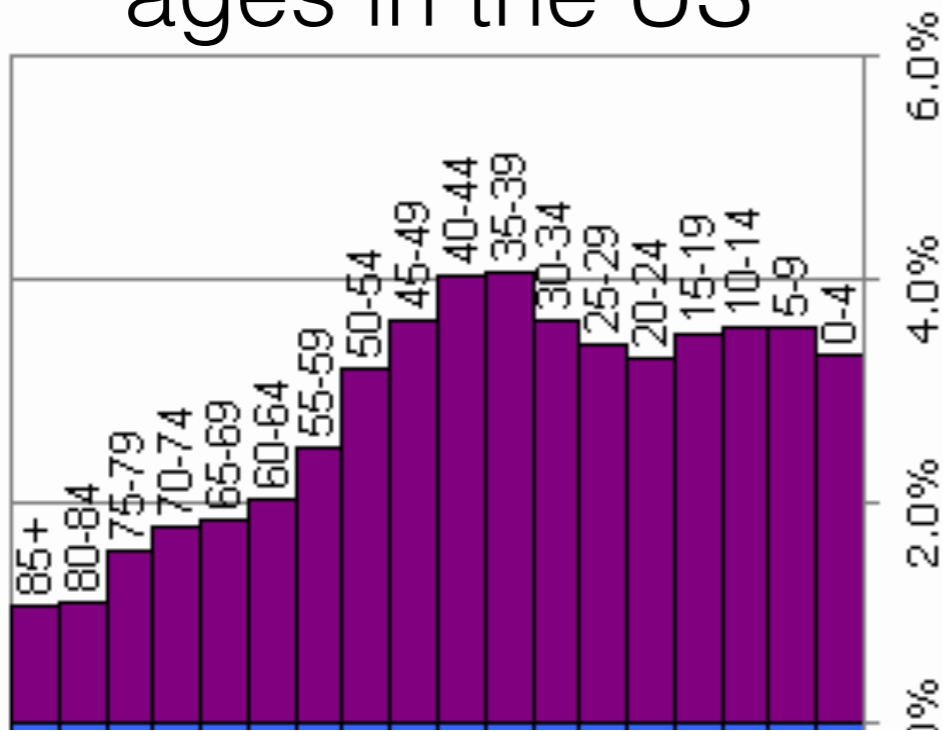
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Thoughts about test statistics?

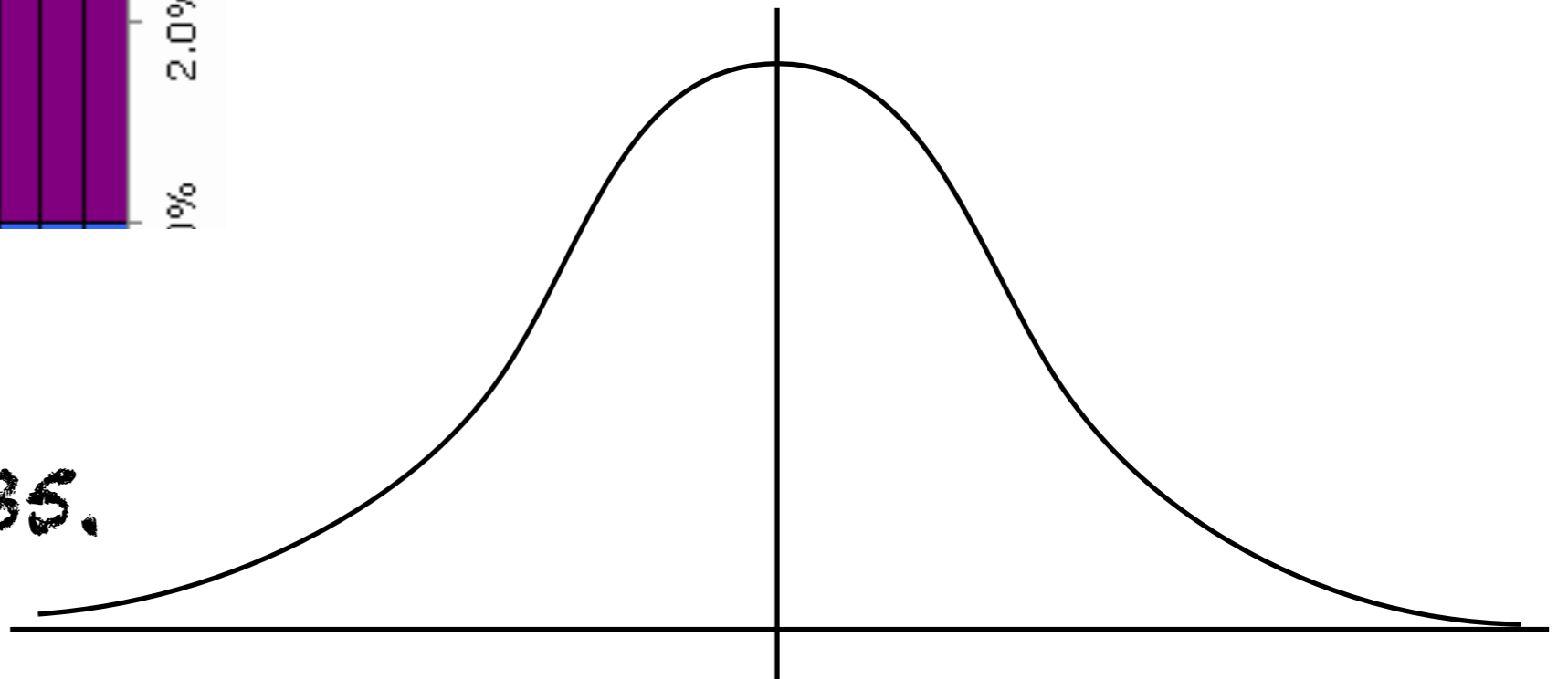
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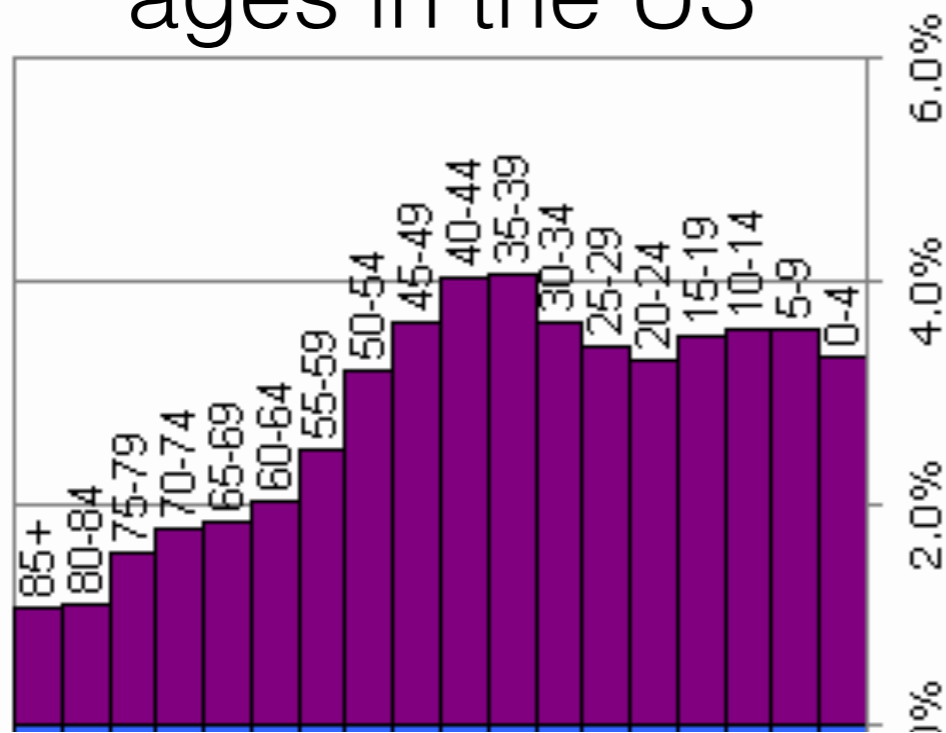
Hypothesis:
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$$z = \frac{\bar{x} - \mu_0}{\sqrt{\frac{\sigma^2}{n}}}$$



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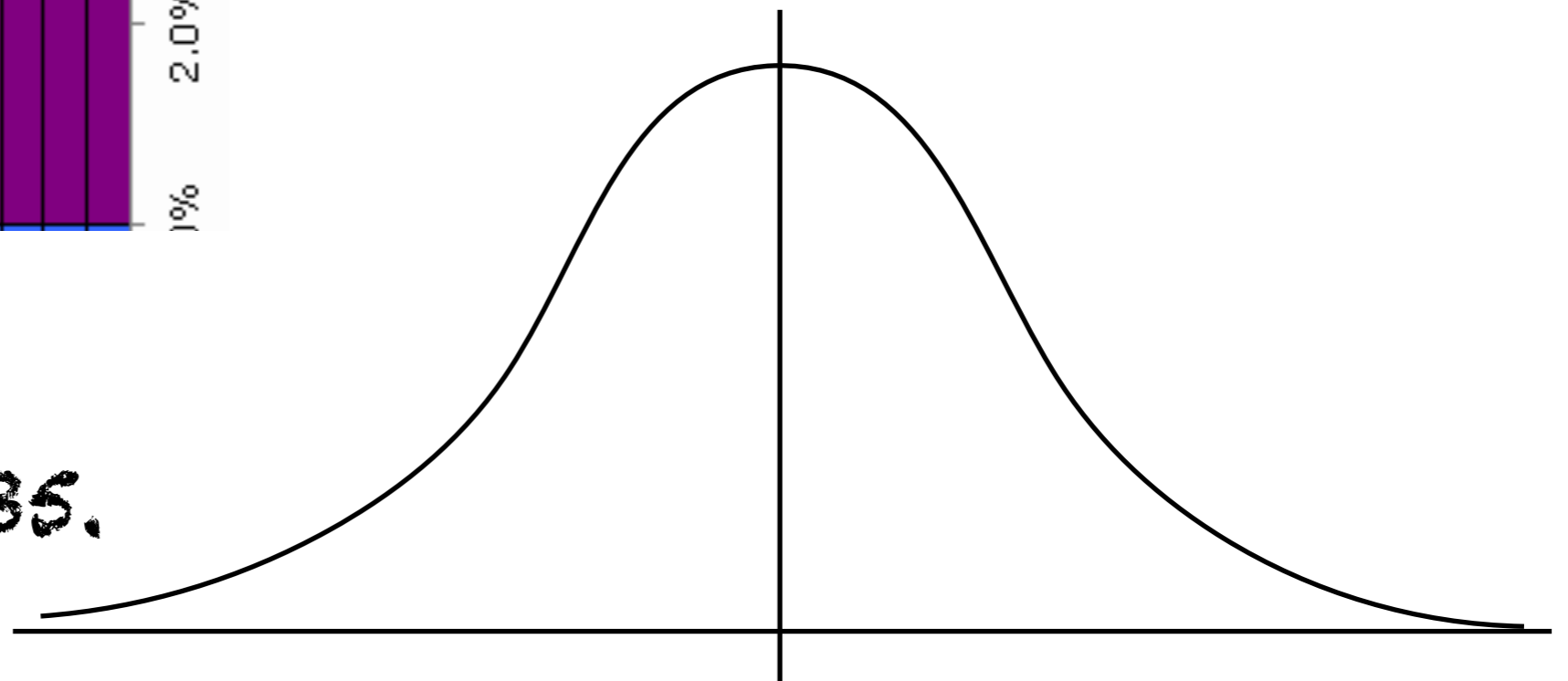
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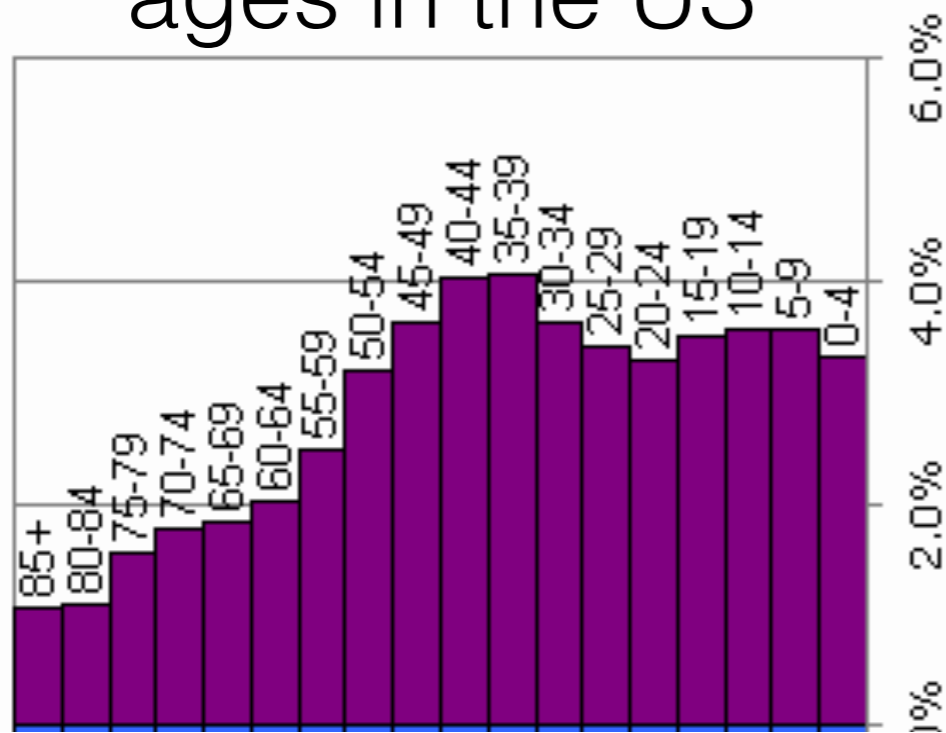
observed

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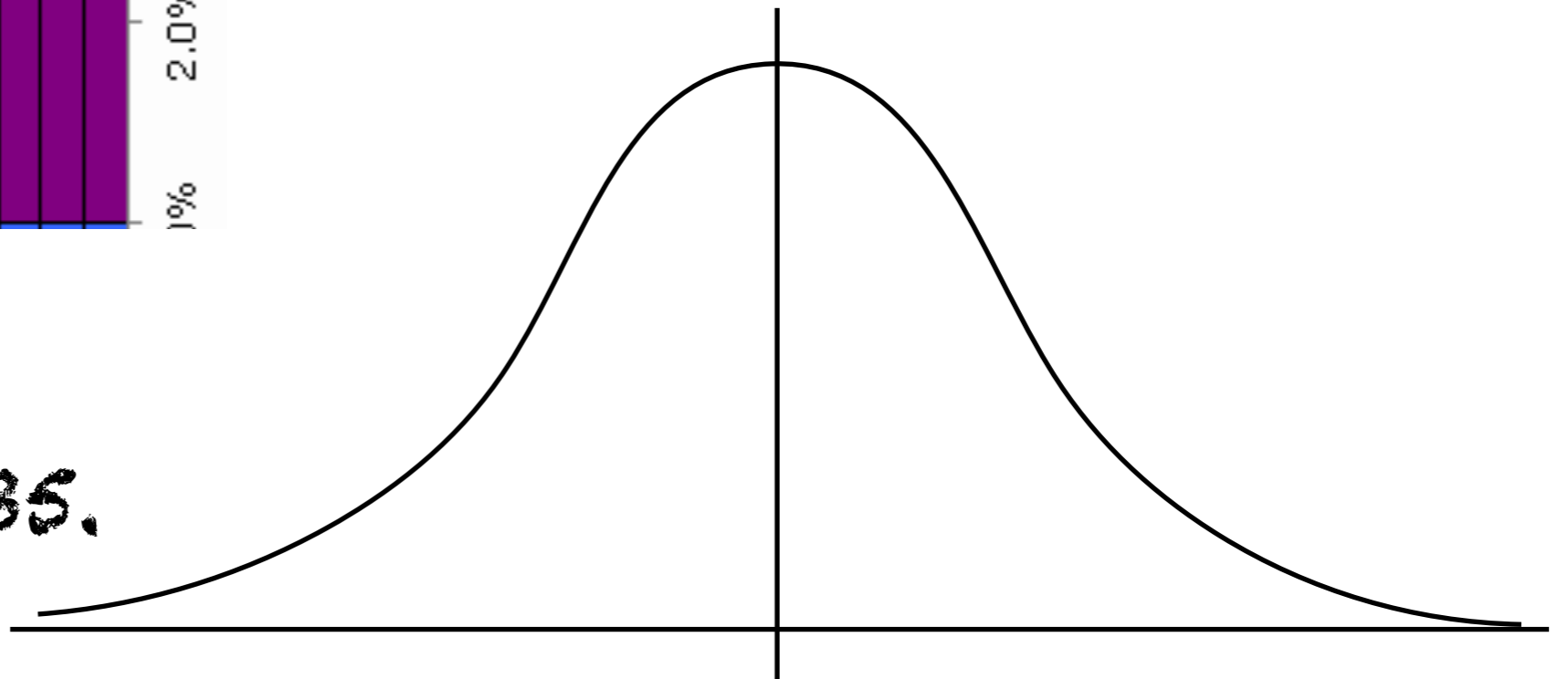
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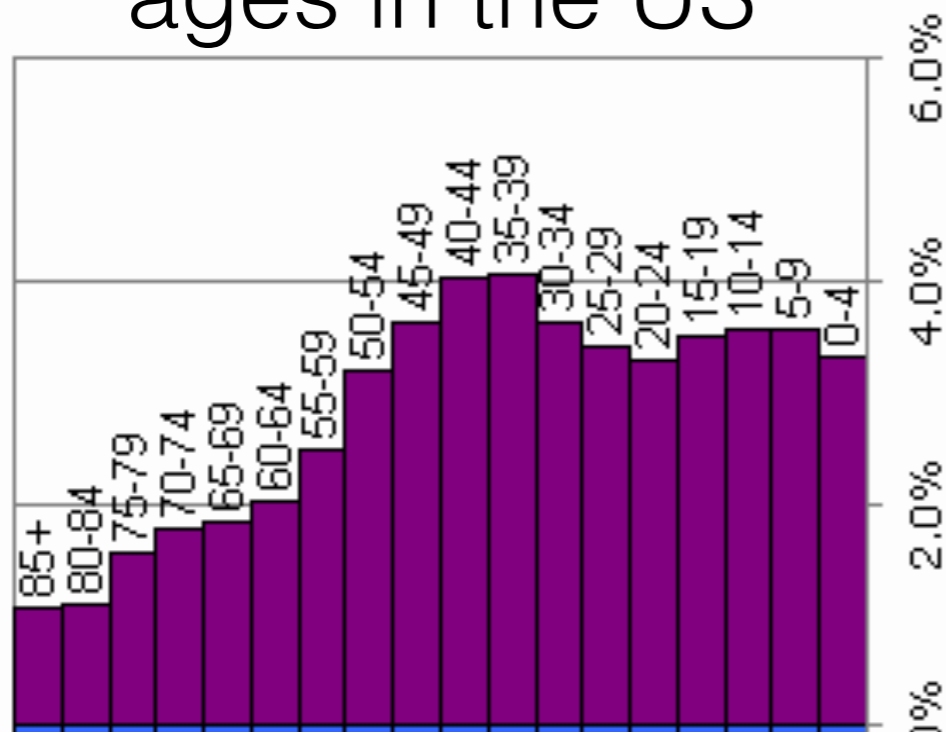
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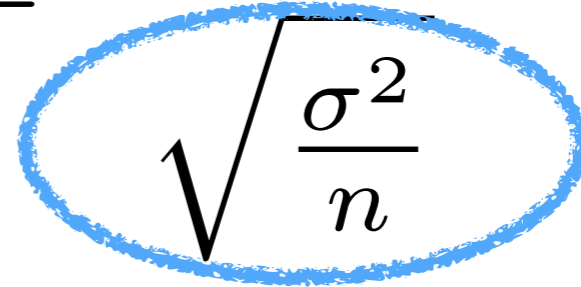
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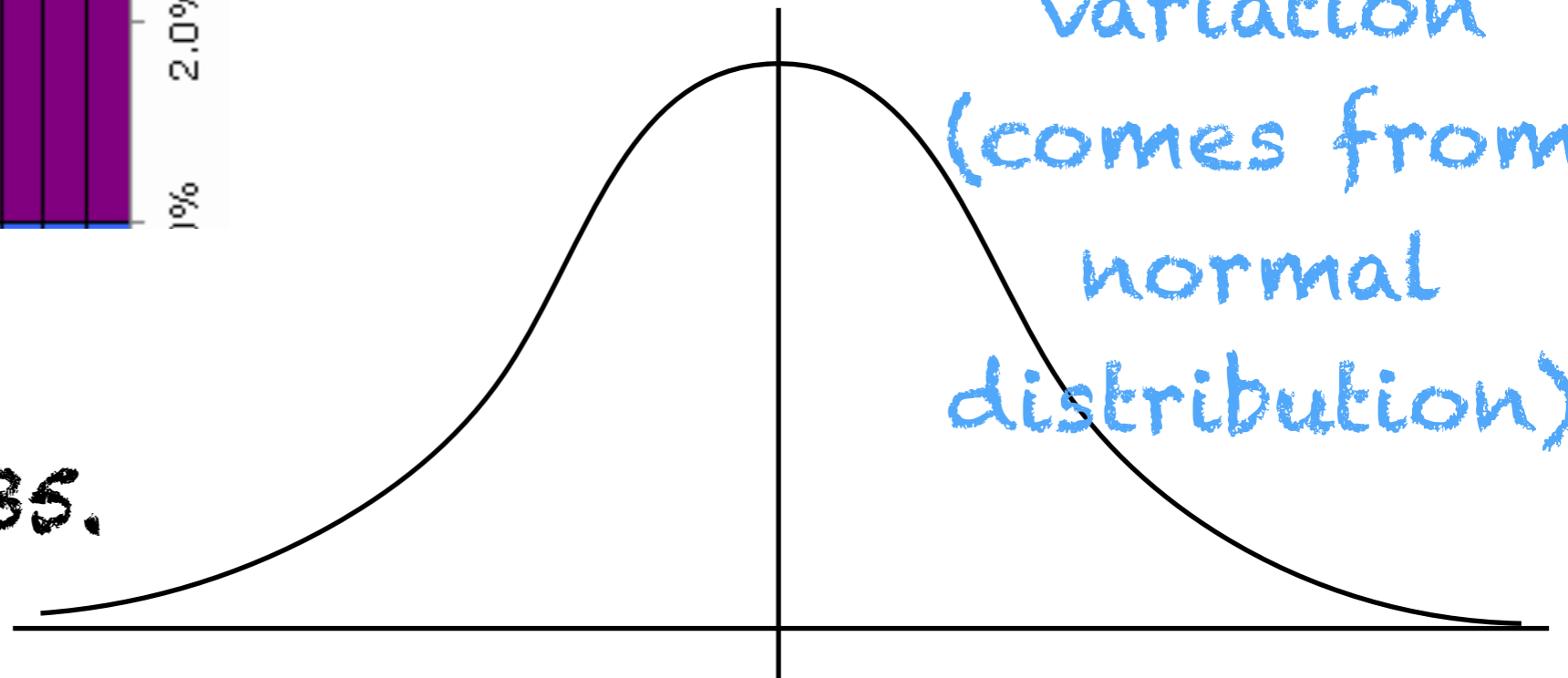


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variation
(comes from
normal
distribution)



Clicker Question!

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Why can we use a normally-distributed test statistic to evaluate mean age of a population?

- a) **Because ages are normally distributed**
- b) **Because the test statistic is a random variable**
- c) **Because of the law of large numbers**
- d) **Because of the central limit theorem**
- e) **The limit does not exist!**

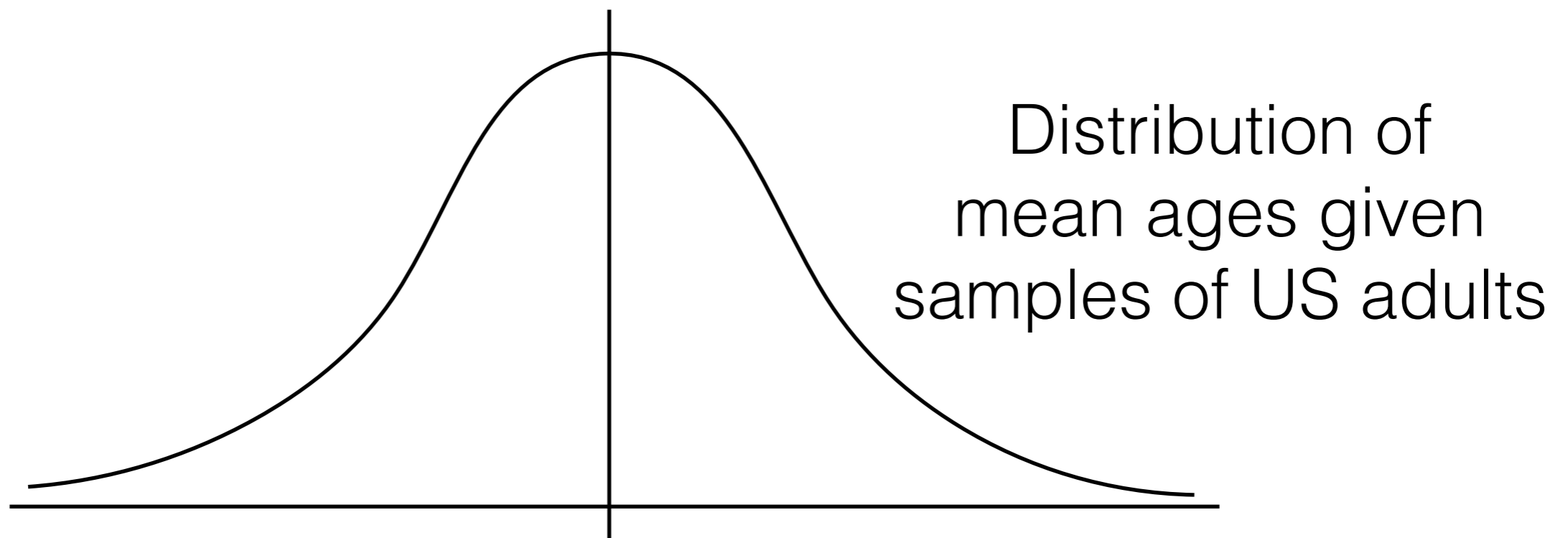
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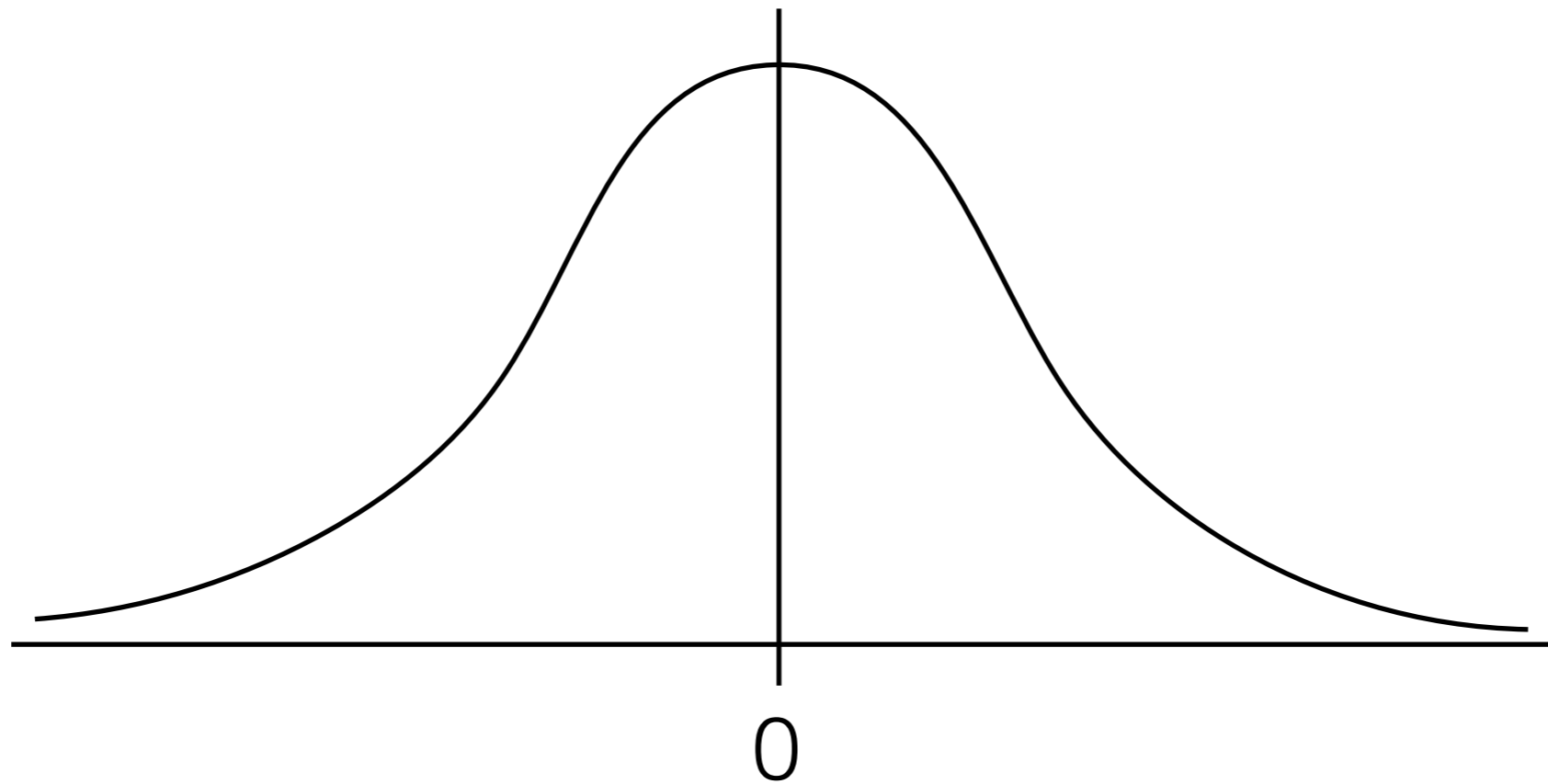
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Standard Normal Distr.

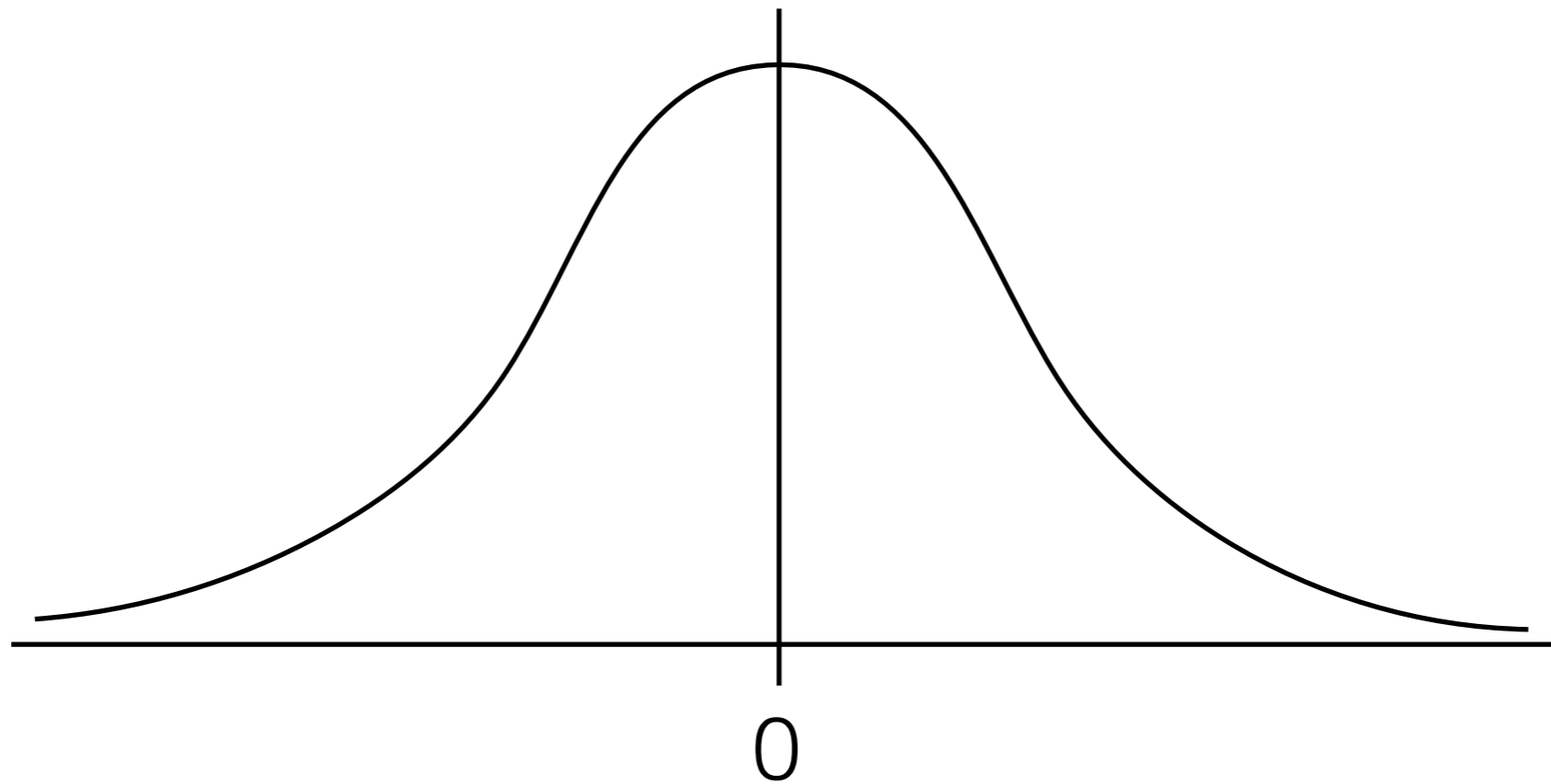
$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$



z = distance from mean in std units

Standard Normal Distr.

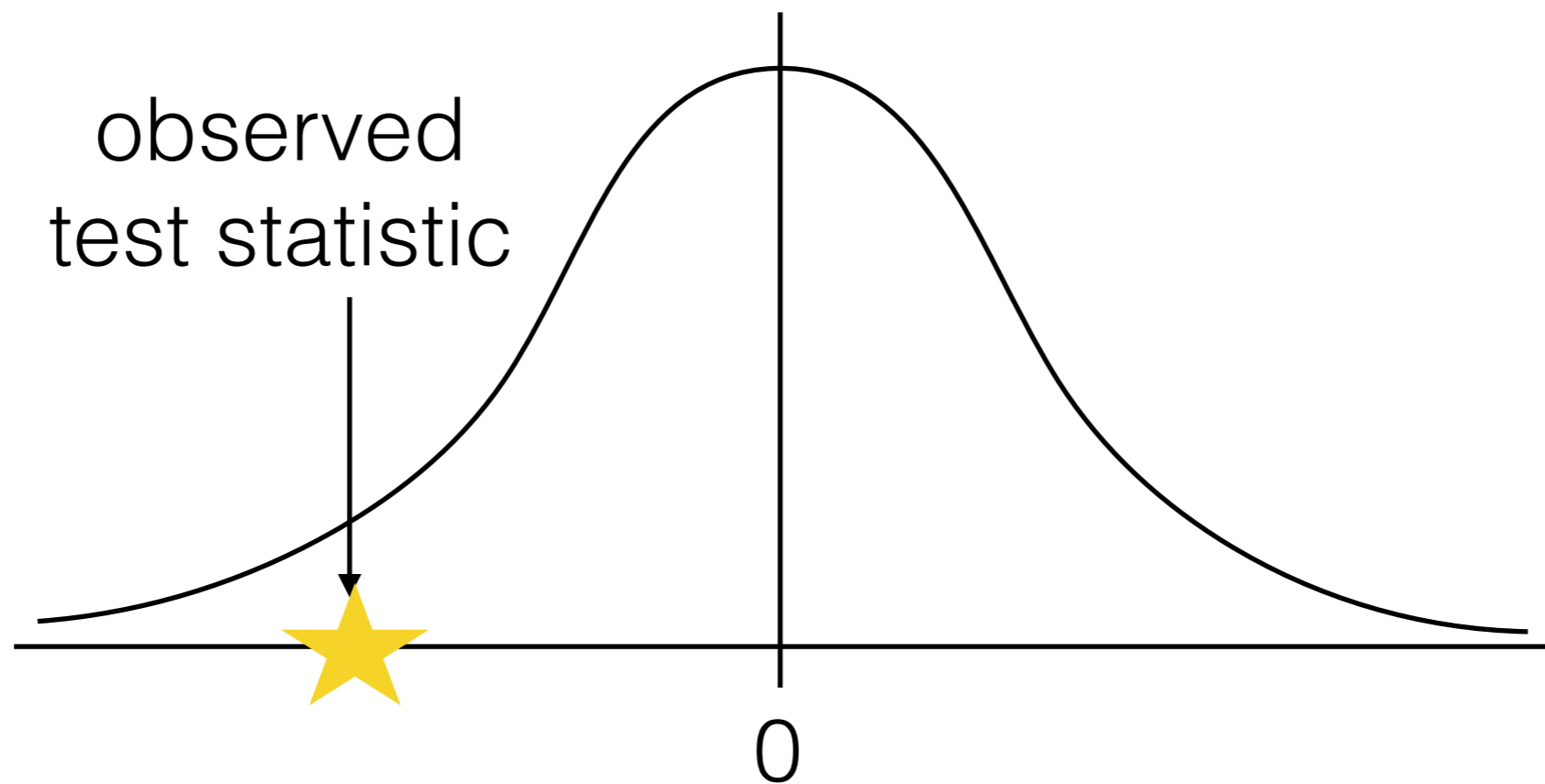
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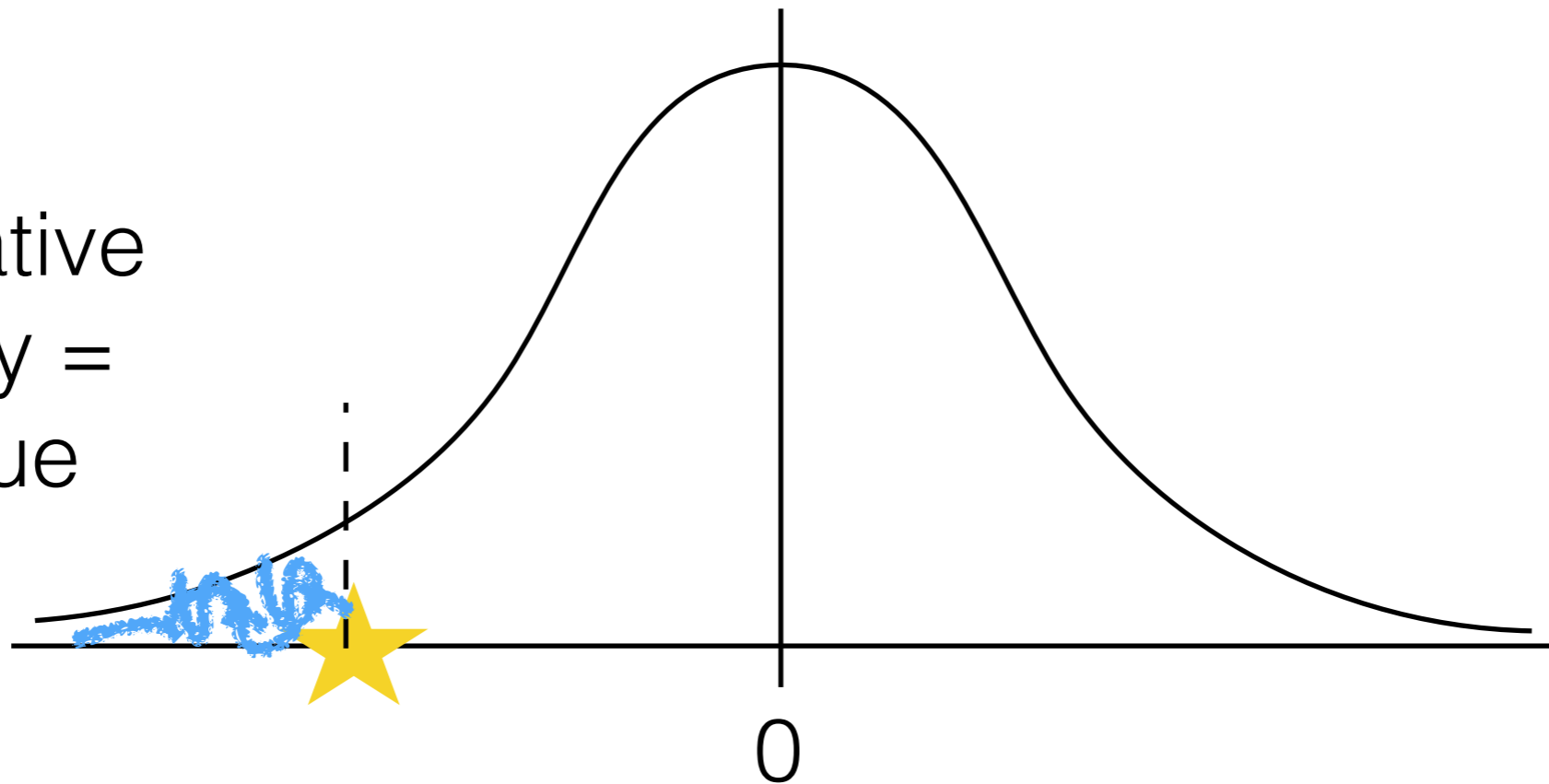


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cumulative
density =
p-value

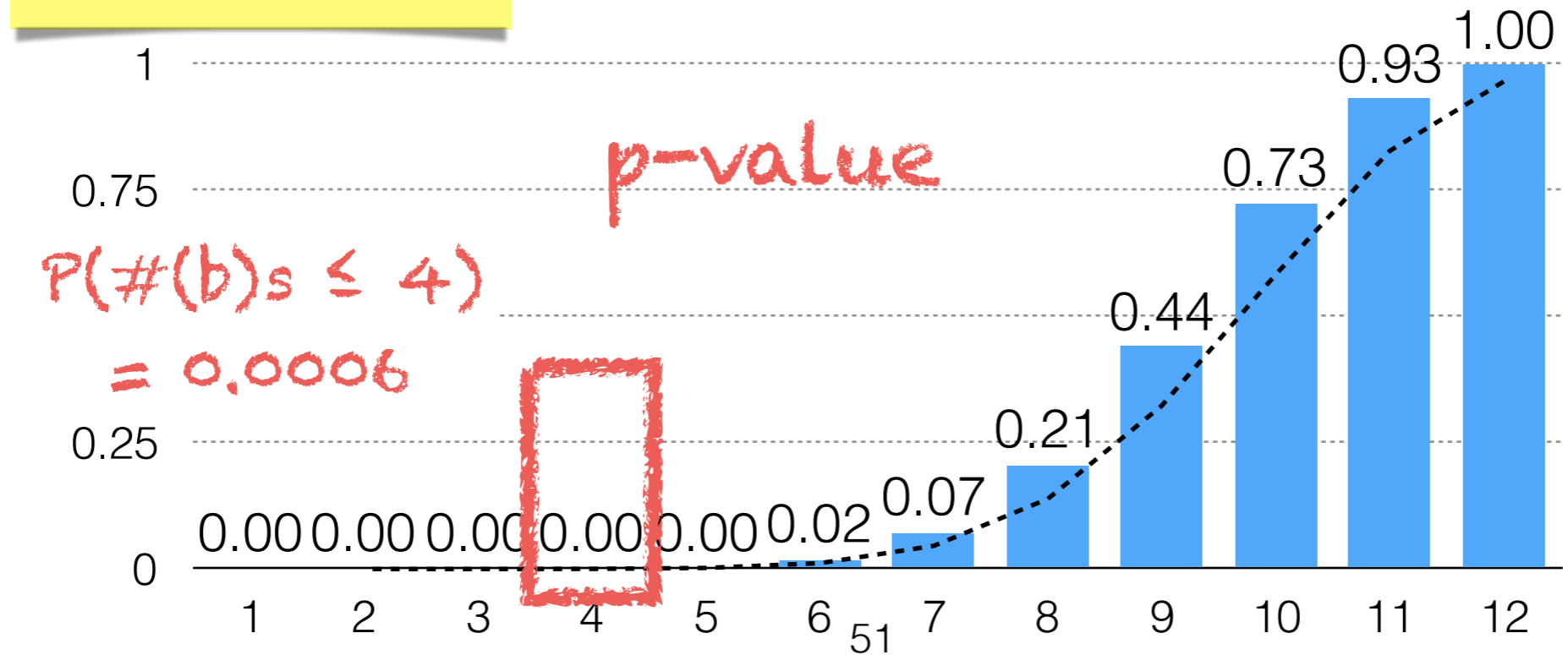
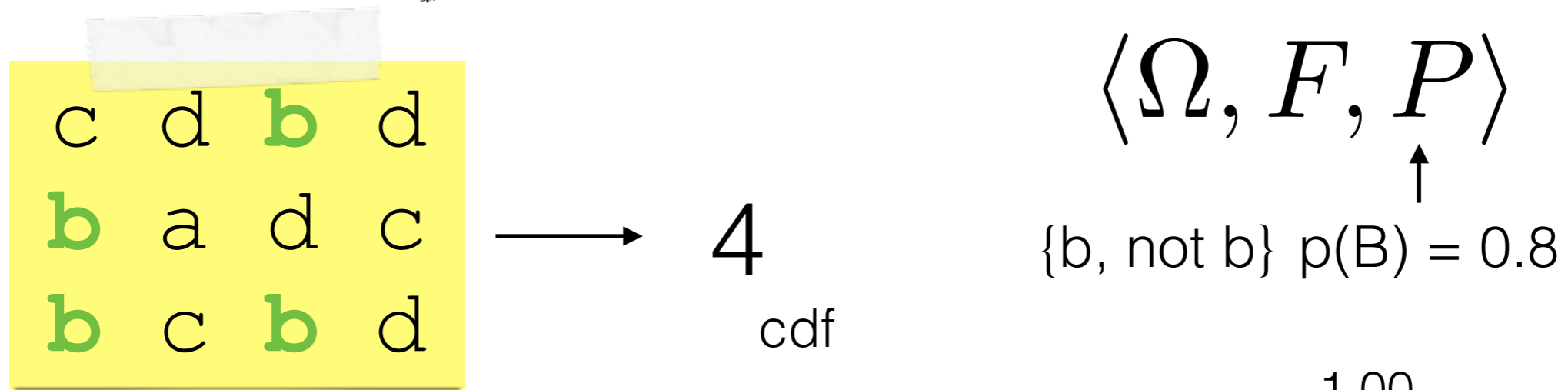


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Detour! α and determining
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“I swear literally like 80% of the answers are just (b)”

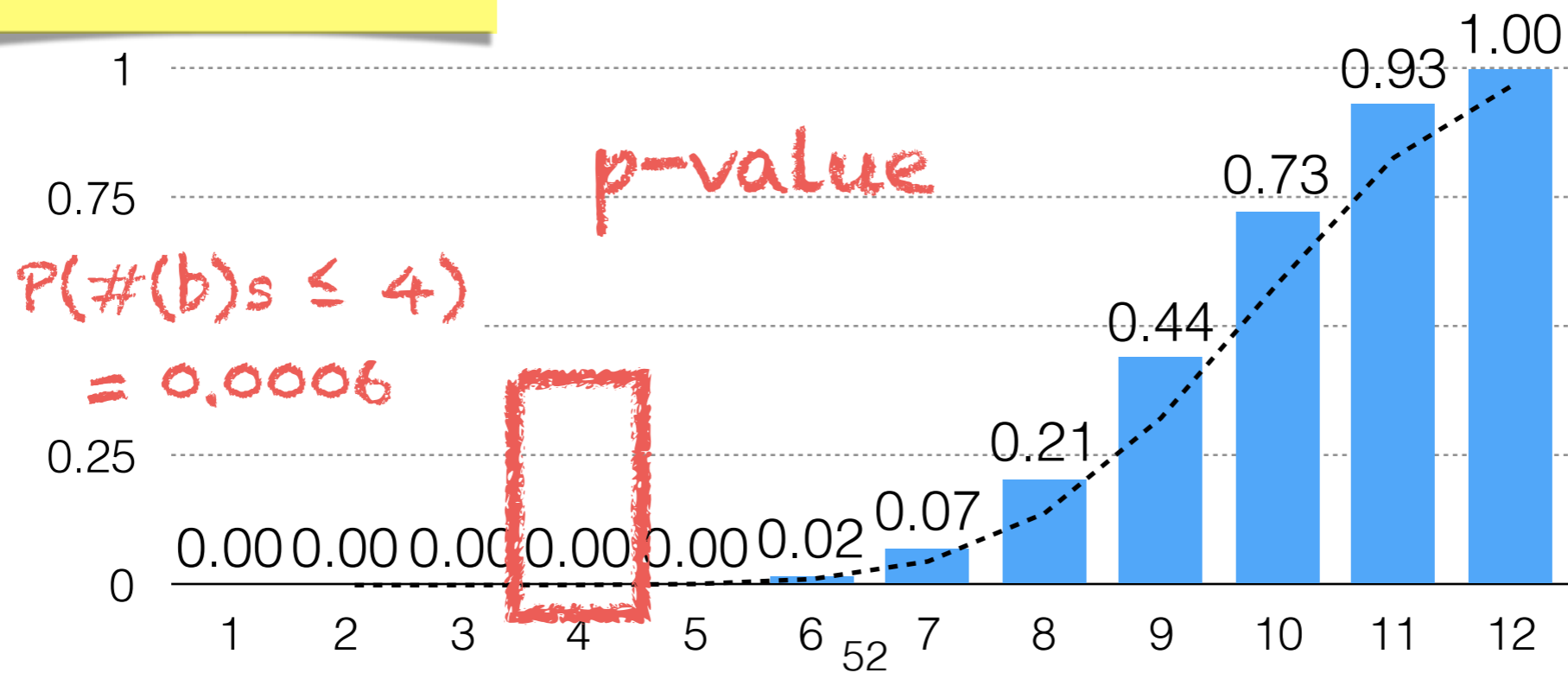


Detour: ... and determining

"assuming the null hypothesis were true, I would only expect to see a test statistic like this 0.0006% of the time. How surprising!"

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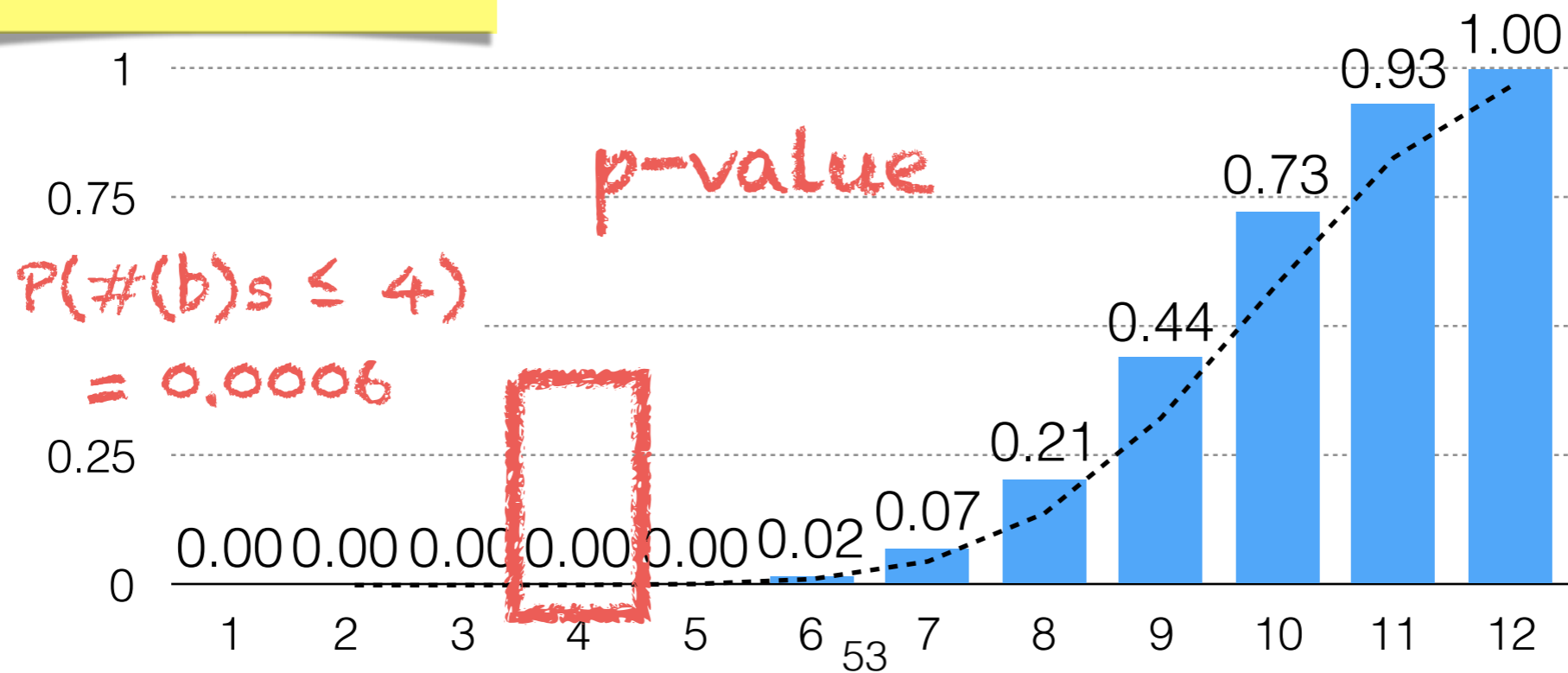


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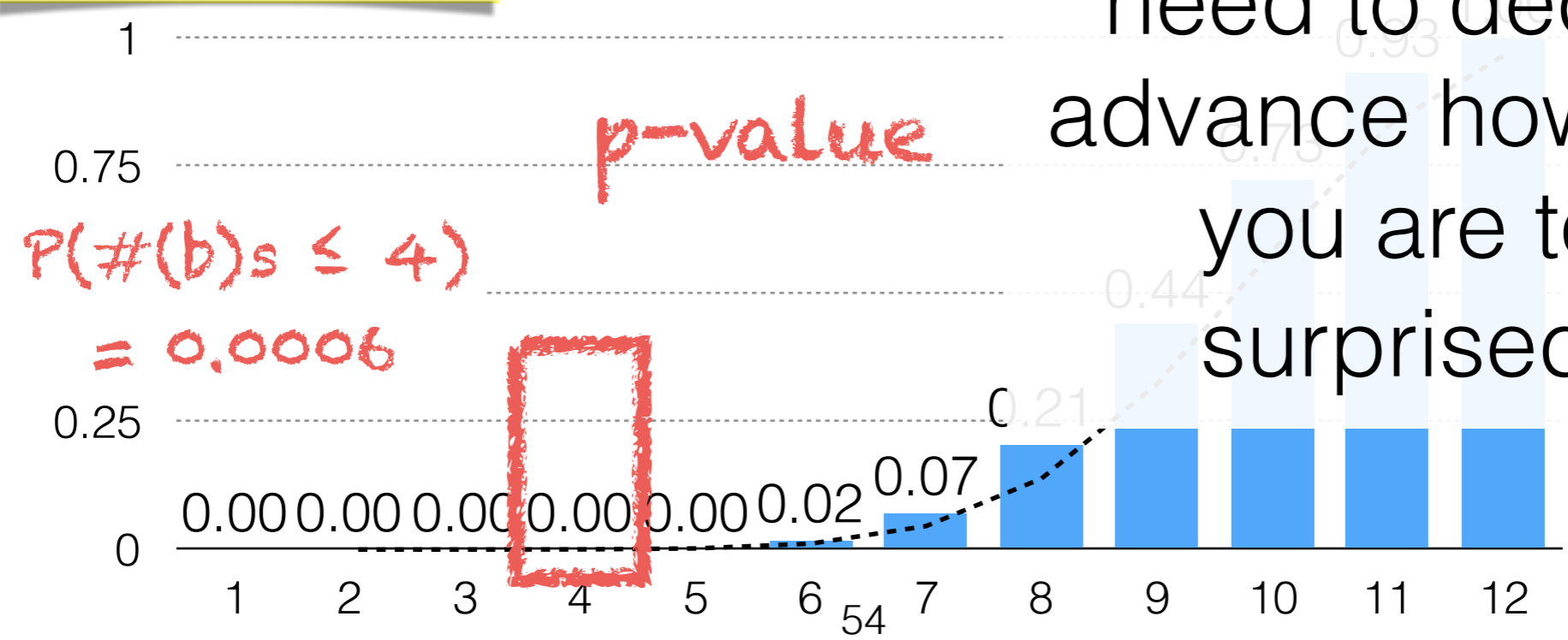


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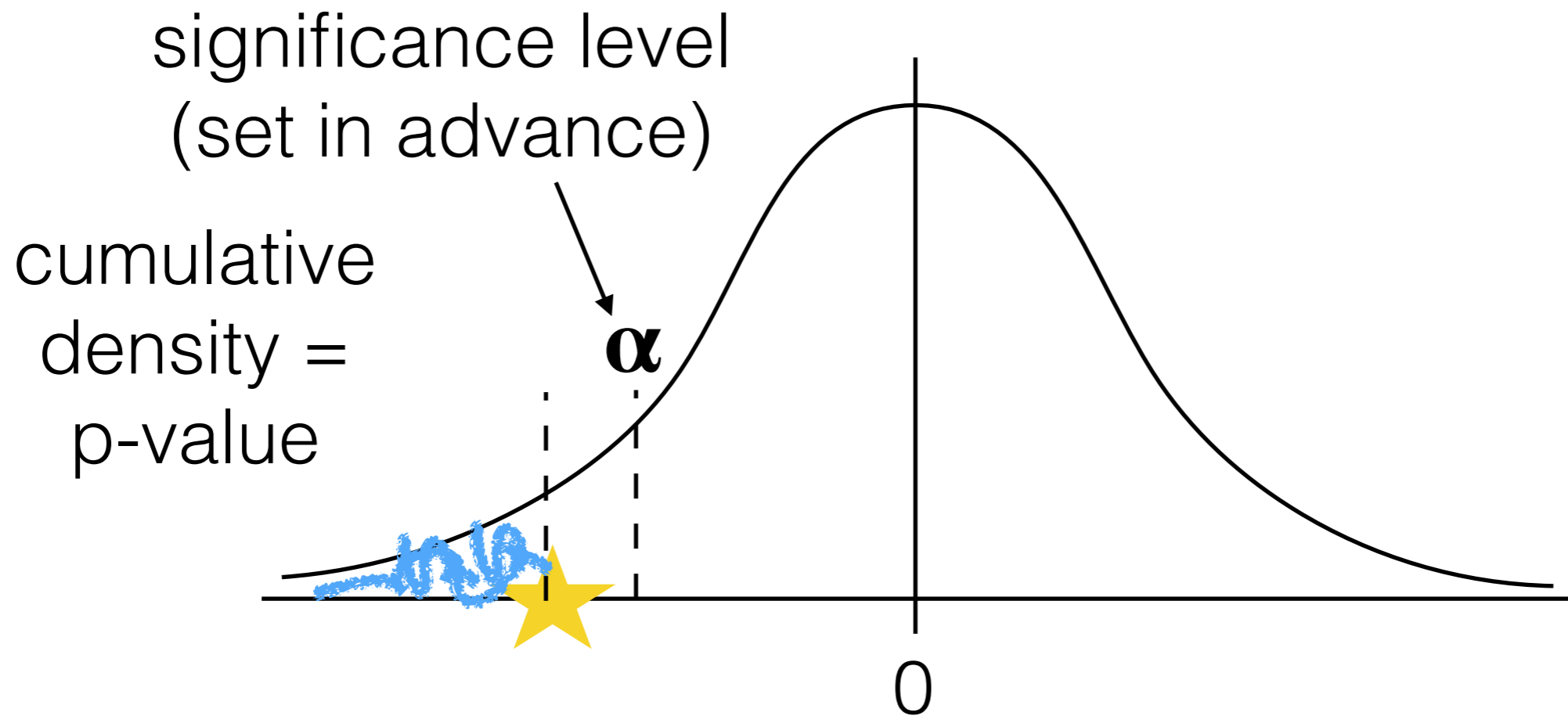
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need to decide in advance how willing you are to be surprised (α)

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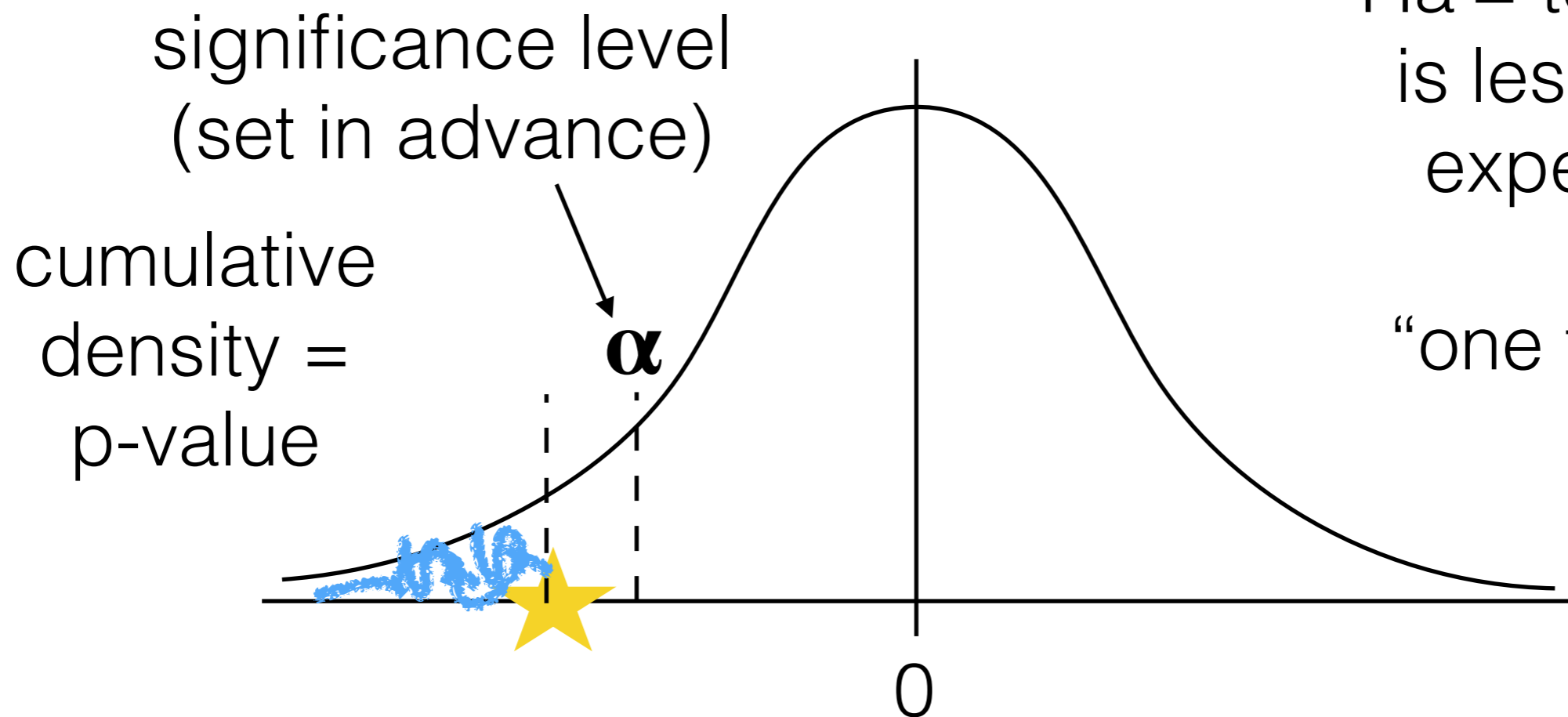


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Ha = test stat
is less than
expected
“one tailed”

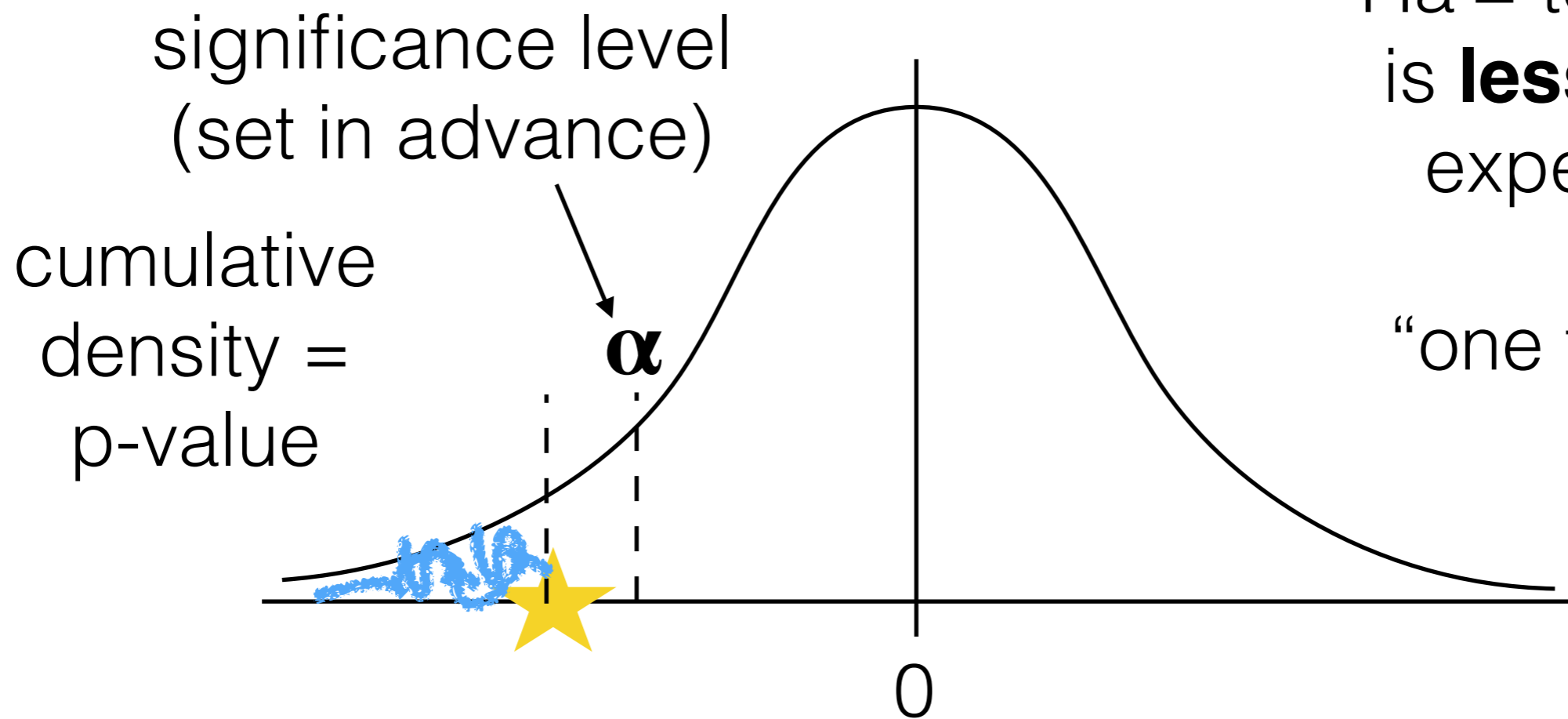


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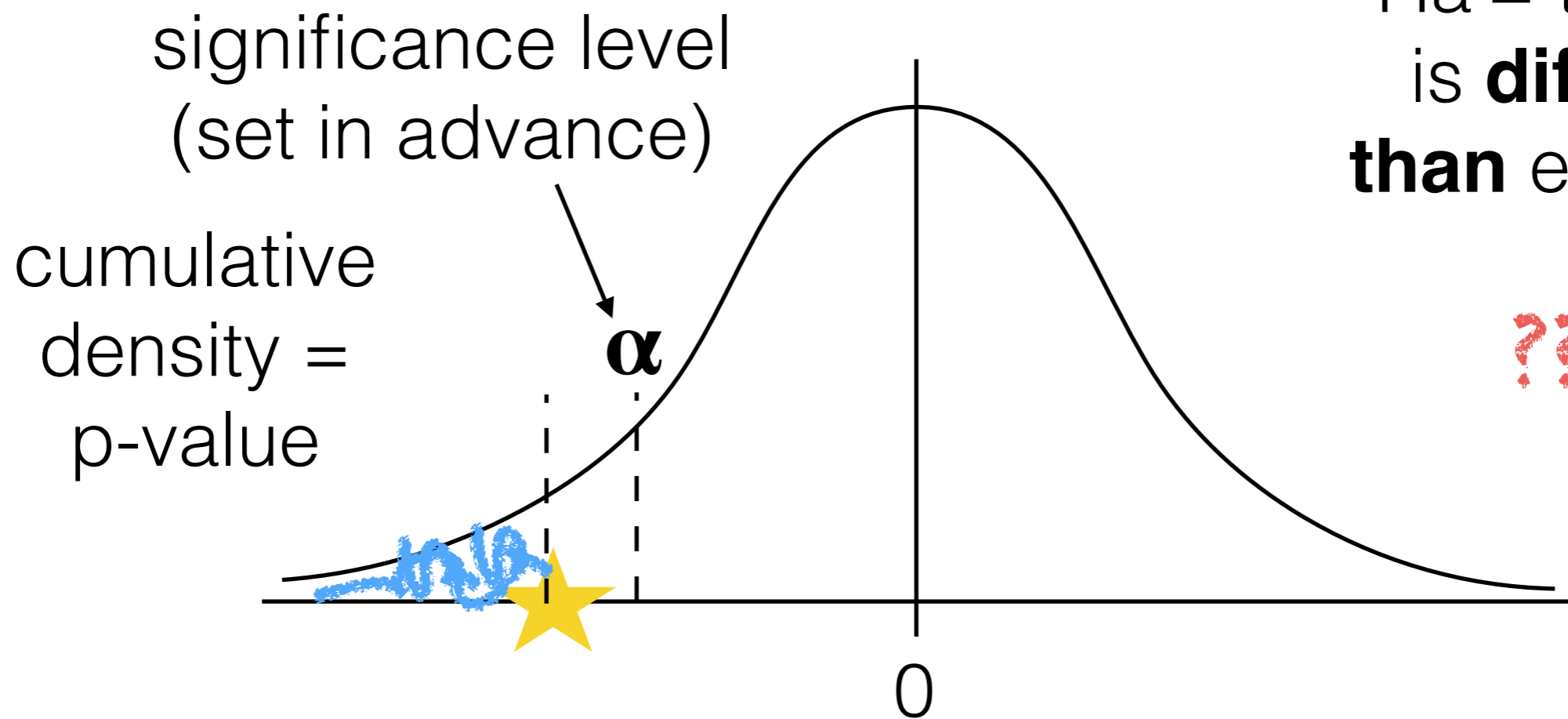
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Ha = test stat
is **different**
than expected

???

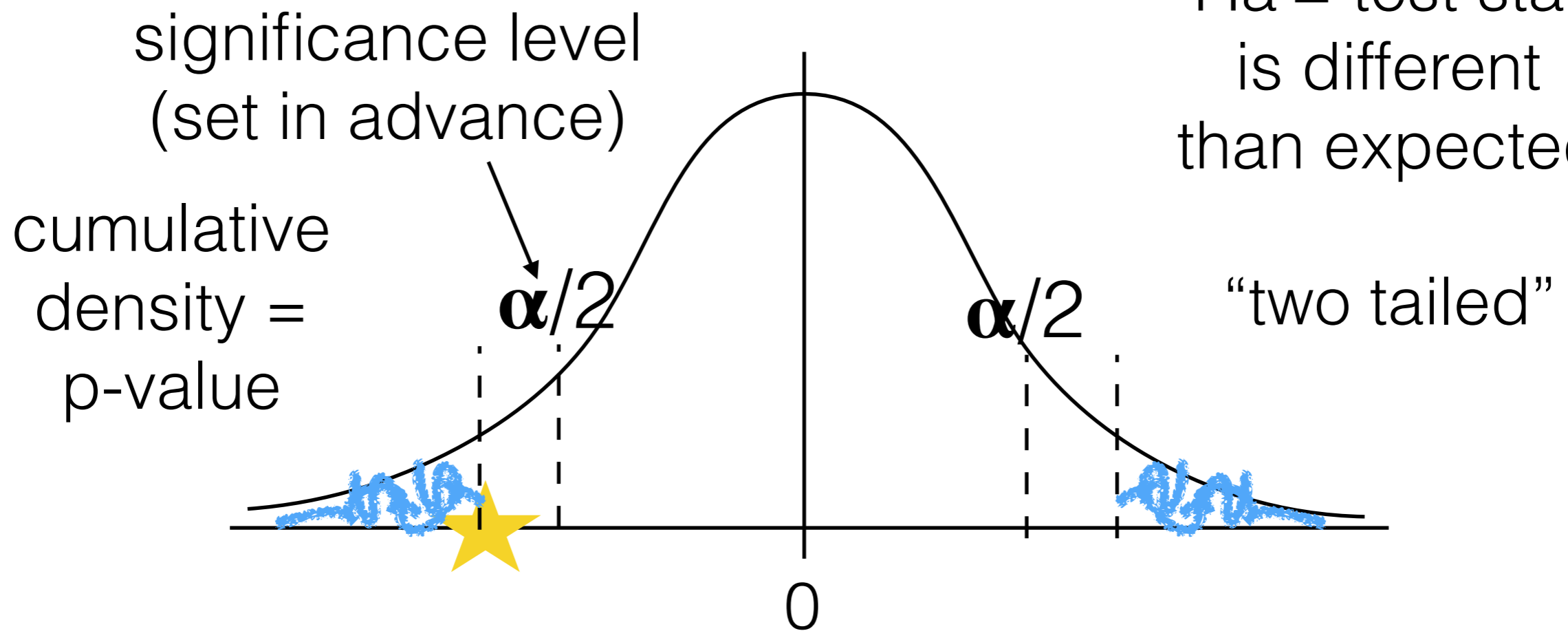


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H_a = test stat
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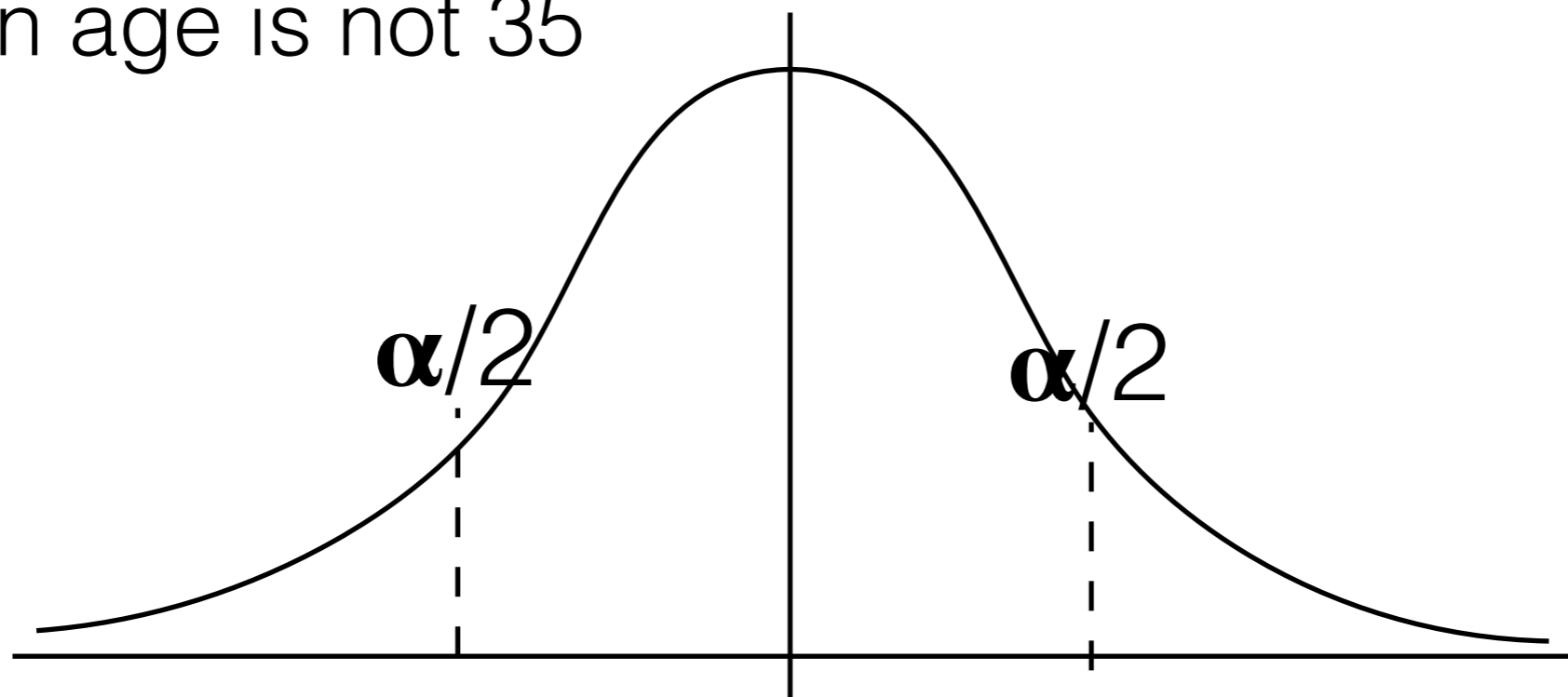
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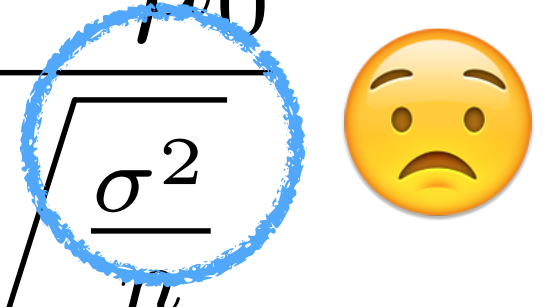
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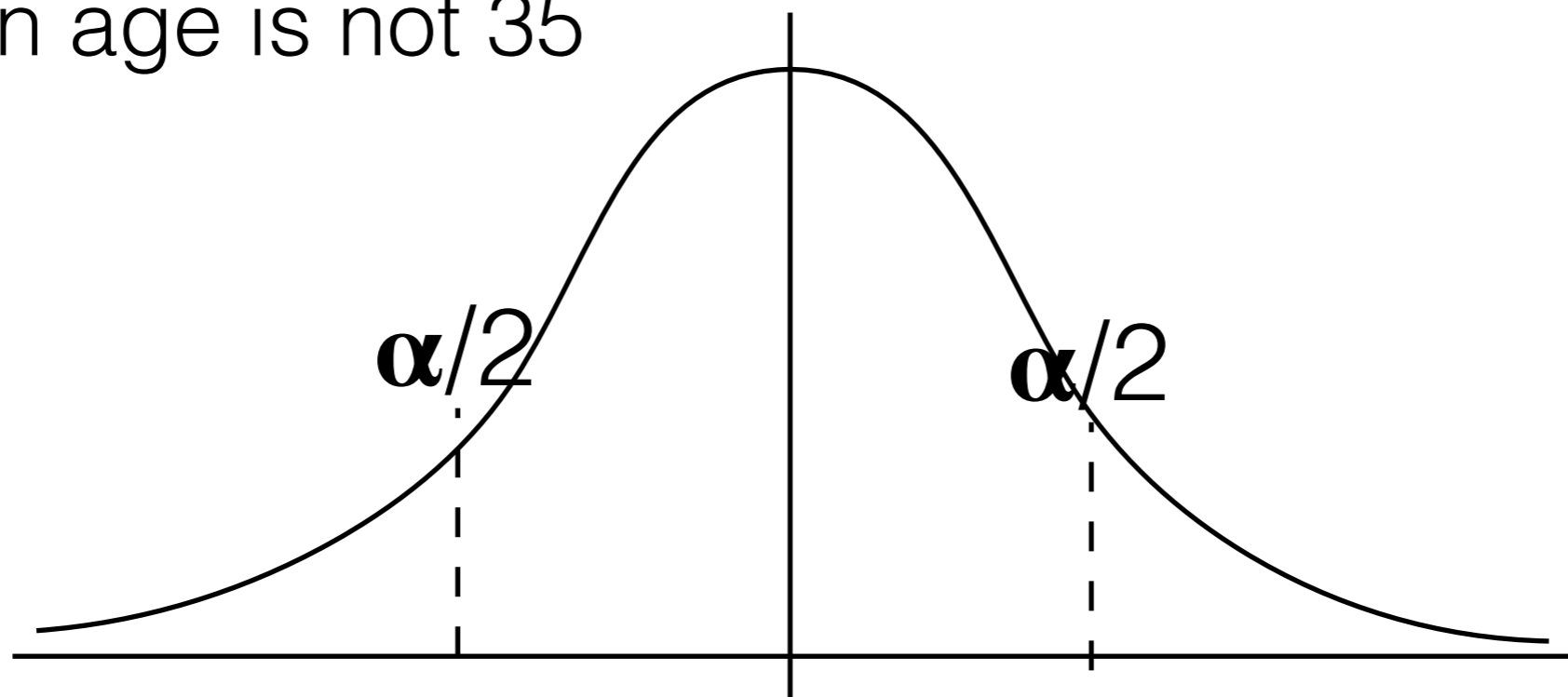


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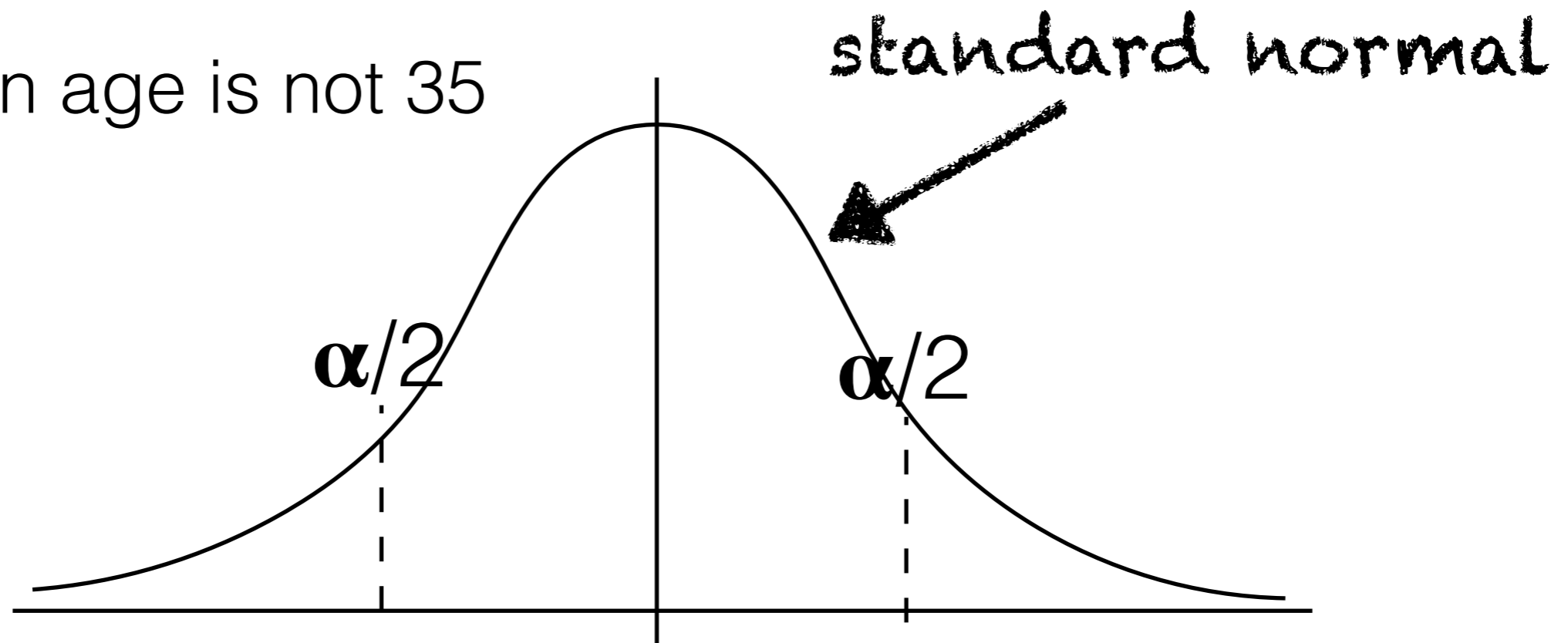


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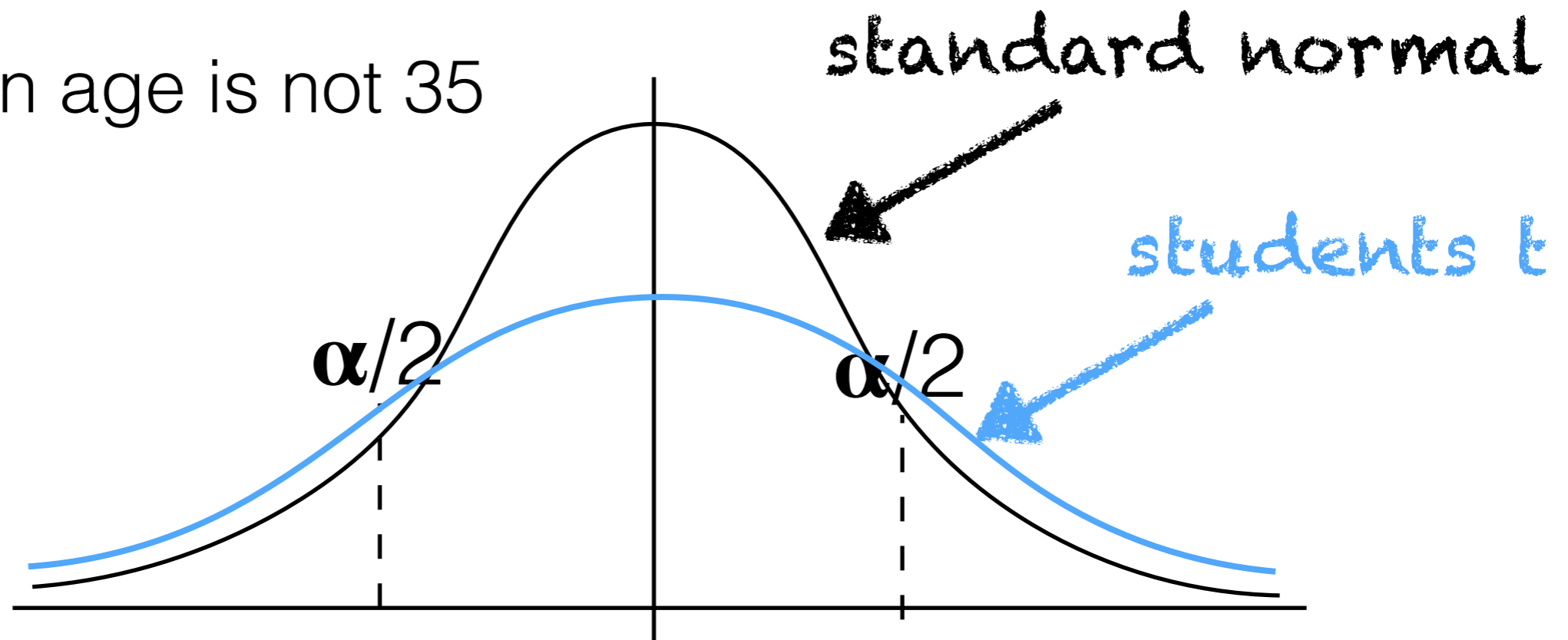


Test for population means

$$z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

H_0 = mean age is 35

H_a = mean age is not 35



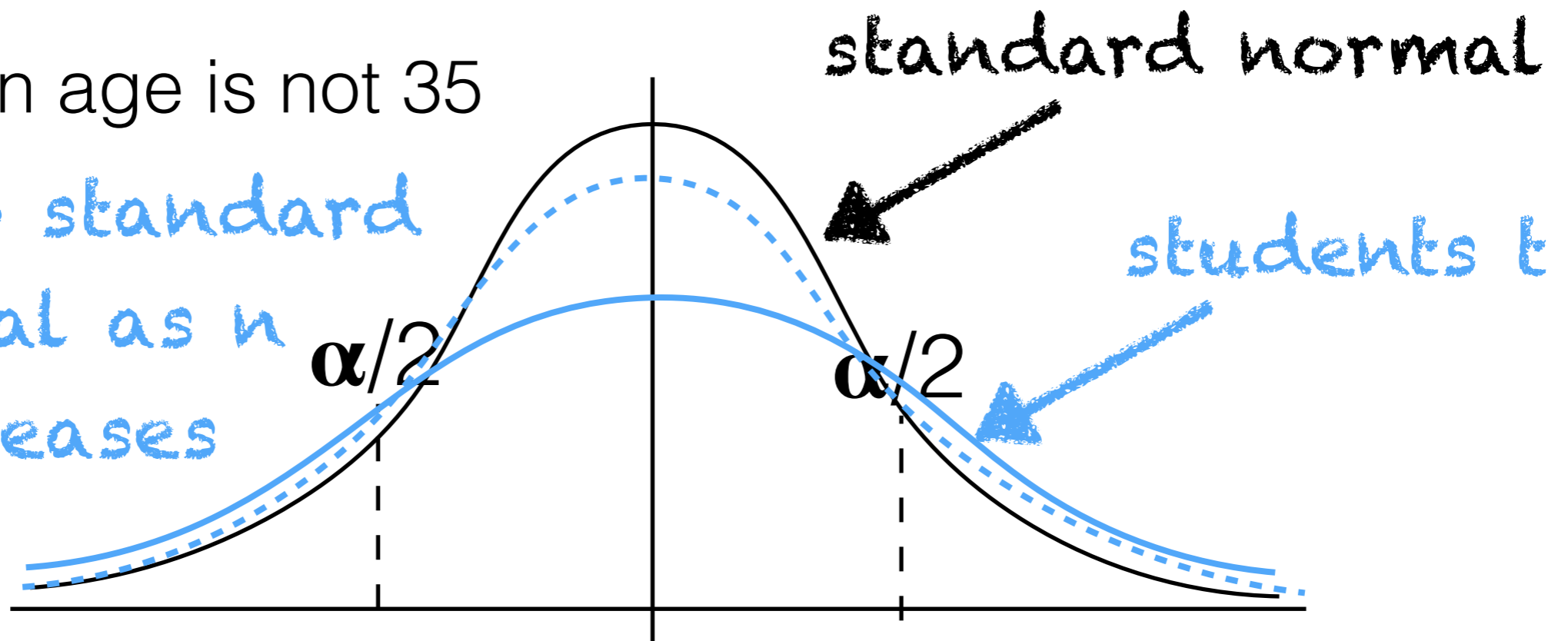
Test for population means

$$z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

H_0 = mean age is 35

H_a = mean age is not 35

closer to standard normal as n increases



Is there evidence to reject the null hypothesis?

Null Hypothesis: The average grade is 85%.

Grades

90

92

80

87

98

78

Clicker Question!

Clicker Question!

What is the mean?

Null Hypothesis: The average grade is 85%.

Grades

90

92

80

87

98

78

a) 85

b) 87.5

c) 90

d) 92.5

Clicker Question!

What is the mean?

Null Hypothesis: The average grade is 85%.

Grades

90

92

80

87

98

78

a) 85

b) 87.5

c) 90

d) 92.5

Clicker Question!

What is the standard error?

Null Hypothesis: The average grade is 85%.

Grades

90

92

80

87

98

78

a) $\sqrt{47.25}$

b) $\sqrt{51.3}$

c) $\sqrt{56.7}$

d) $\sqrt{57.25}$

$$s^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2$$

Clicker Question!

What is the standard error?

Null Hypothesis: The average grade is 85%.

Grades

90

92

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d) $\sqrt{57.25}$

$$s^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2$$

Clicker Question!

What is the test statistic?

Null Hypothesis: The average grade is 85%.

$$z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

mean: 87.5
s: 7.5

Grades

90

92

80

87

98

78

a) 0.82

b) 0.85

c) 0.91

d) 0.95

Clicker Question!

What is the test statistic?

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mean: 87.5
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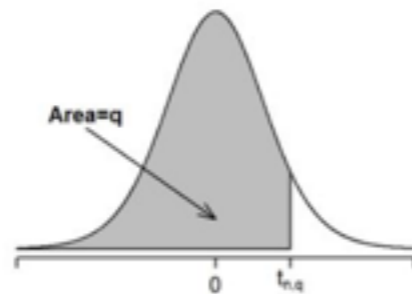
Is there evidence to reject the null hypothesis?

Null Hypothesis: The average grade is 85%.

$$\bar{x} = 82\%$$

Quartiles of the t Distribution

The table gives the value of $t_{n,q}$ - the q th quantile of the t distribution for n degrees of freedom



	$q = 0.6$	0.75	0.9	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$n = 1$	0.3249	1.0000	3.078	6.314	12.706	31.821	63.657	127.321	318.309	636.619
2	0.2887	0.8165	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.2767	0.7649	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.2707	0.7407	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.2672	0.7267	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.2648	0.7176	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959

- a) 0.82
- b) 0.85
- c) 0.91
- d) 0.95

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$$\bar{x} = 82\%$$

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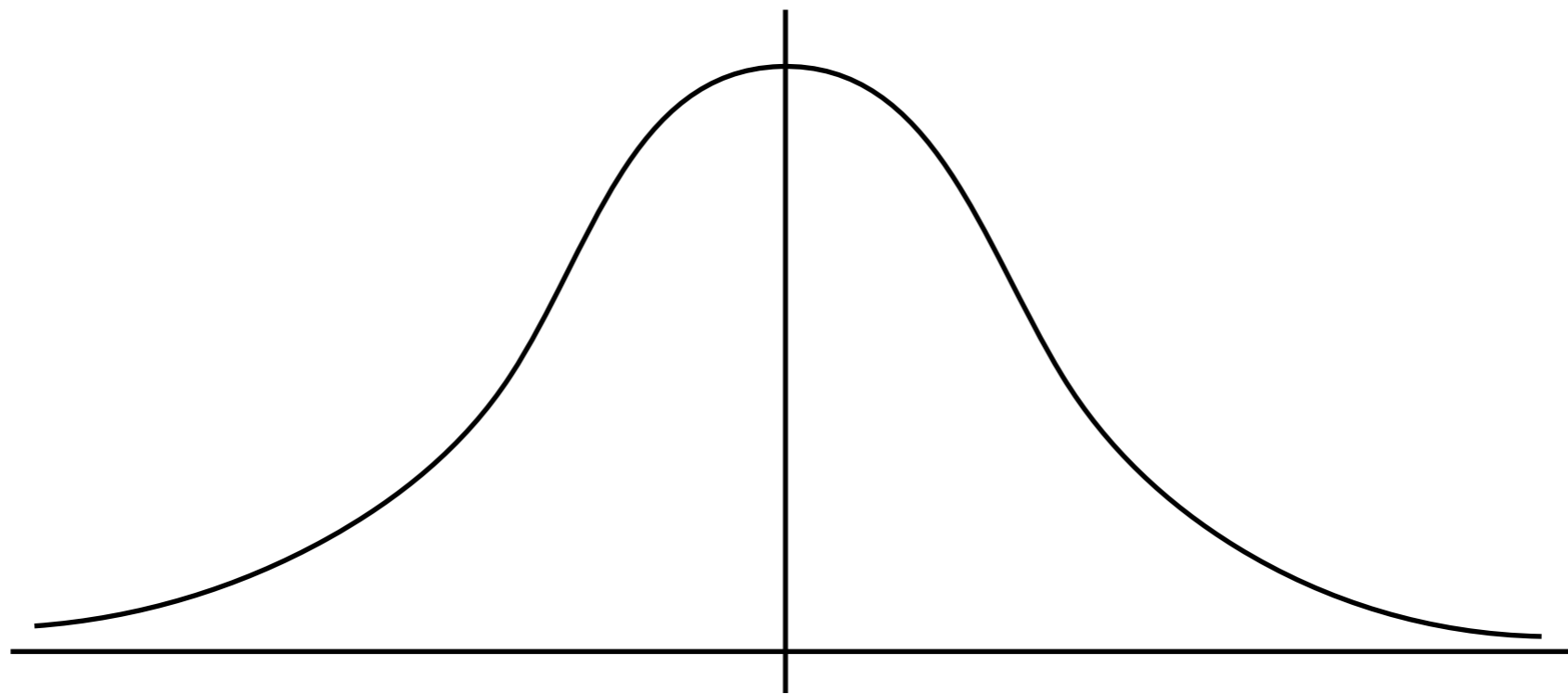
ttest_1samp

n	$q = 0.6$	0.75	0.9	0.95	0.975				0.999	0.9995
1	0.3249	1.0000	3.078	6.314	12.706	31.821	63.657	318.309	636.619	
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- a) 0.82
- b) 0.85
- c) 0.91
- d) 0.95

Test for difference in means

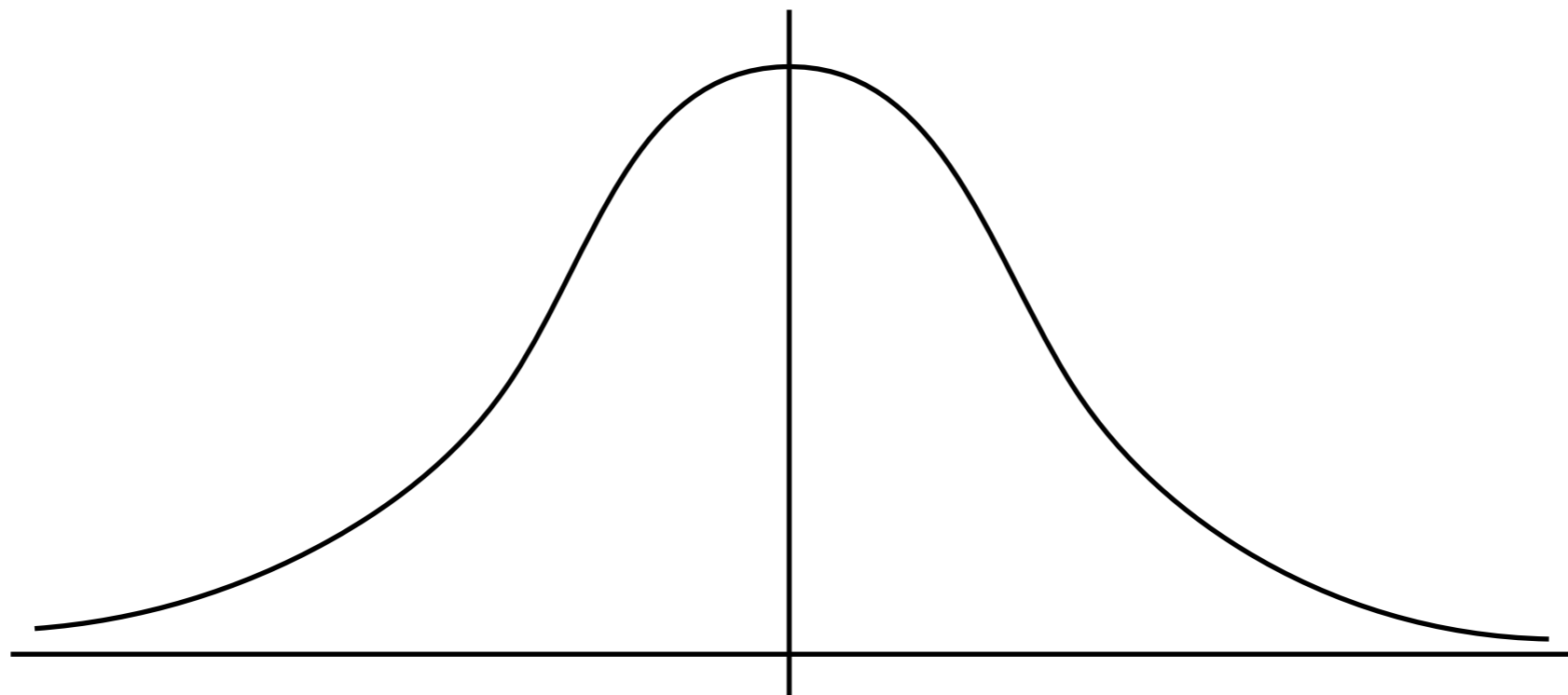
$$z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$



Test for difference in means

$$z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

mean 1 - mean 2

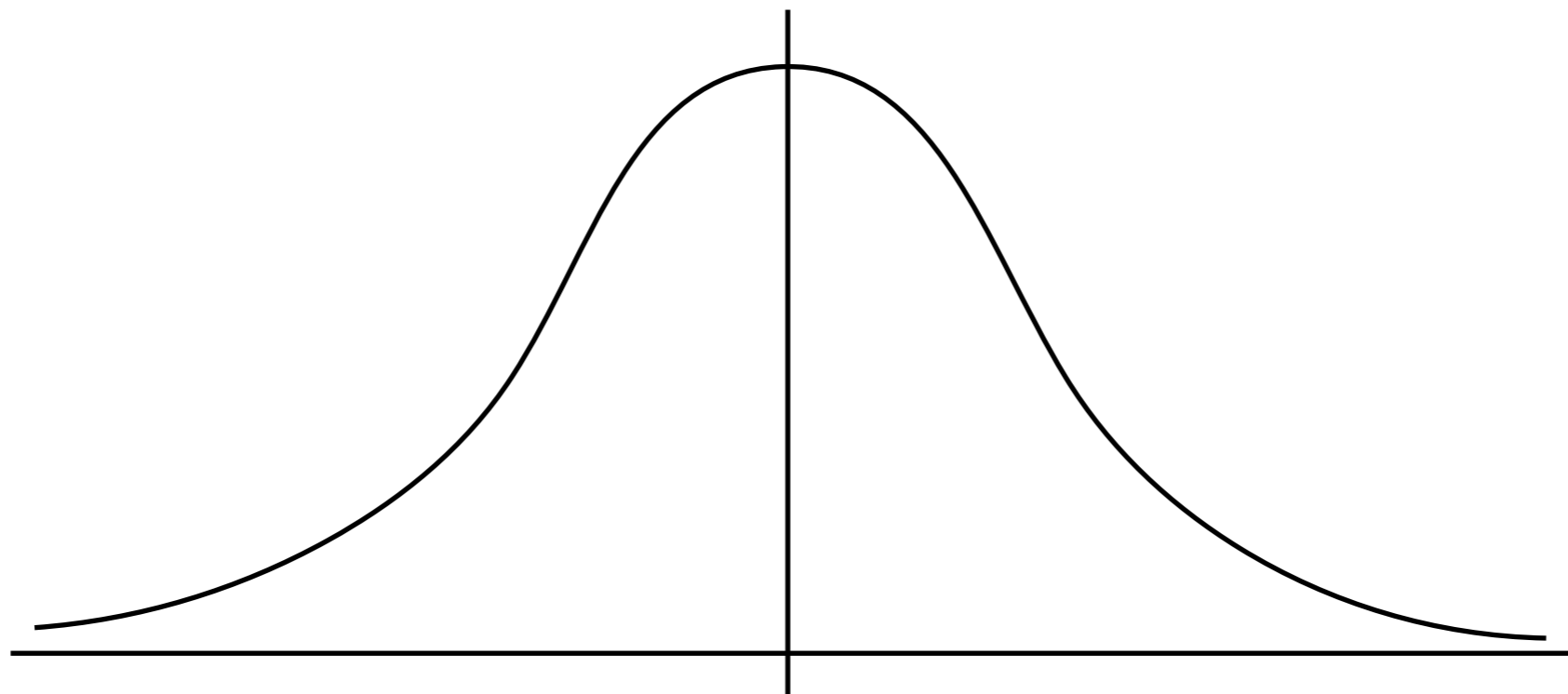


Test for difference in means

$$z = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

mean 1 - mean 2

zero



Is there evidence to reject the null hypothesis?

Null Hypothesis: The average grade is the same as last year.

Alt. Hypothesis: This year's grades are higher.

<u>2019</u>	<u>2020</u>
90	95
92	92
80	83
87	87
98	98
78	75

Is there evidence to reject the null hypothesis?

Null Hypothesis: The average grade is the same as last year.

Alt. Hypothesis: This year's grades are higher.

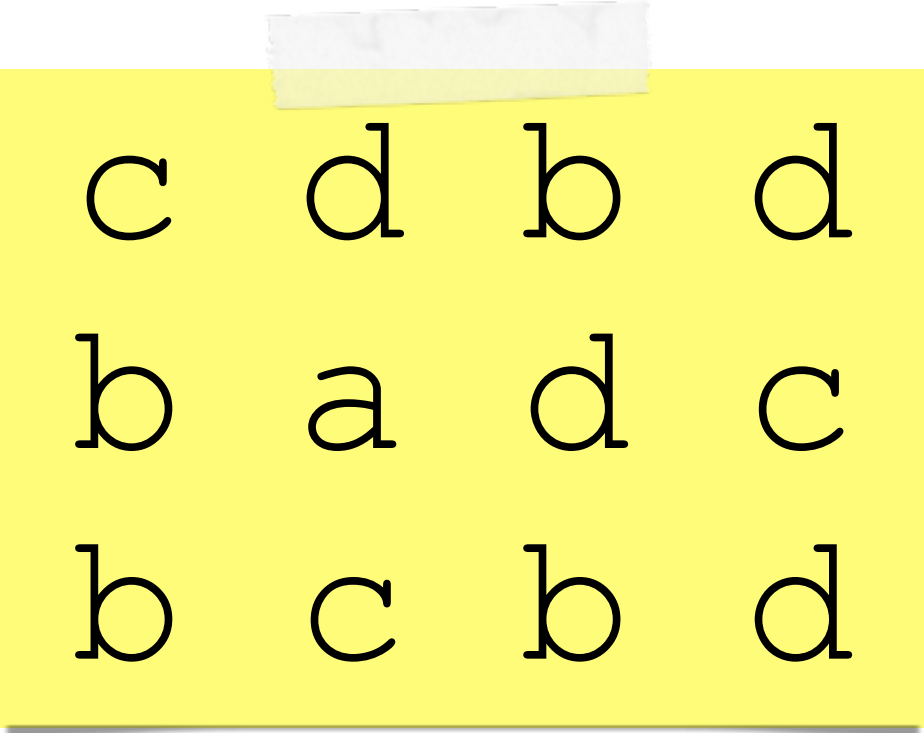
<u>2019</u>	<u>2020</u>
90	95
92	92
80	83
87	87
98	98
78	75

ttest_ind

Some tests you are likely to use

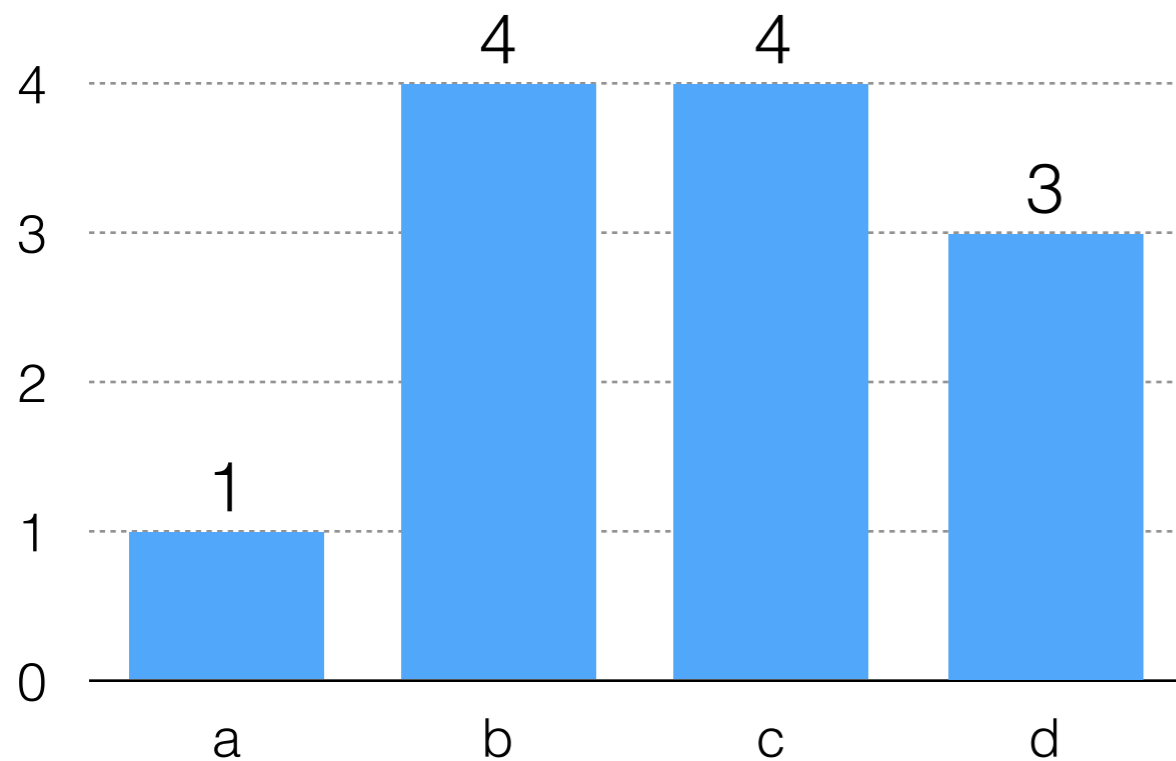
- t-test: difference of means; is the average value of some feature different between two populations
 - e.g. are men taller than women, are blue states more populated than red states, do CS work harder than other majors (::rolling_eyes::)
- **chi-squared test: difference in frequencies of a categorical variable; is the distribution of some feature uniform across groups**
 - e.g. do neighborhoods differ in terms of music preferences features; do college majors differ in terms of sociodemographic features

Are the answers to my
clicker questions random?

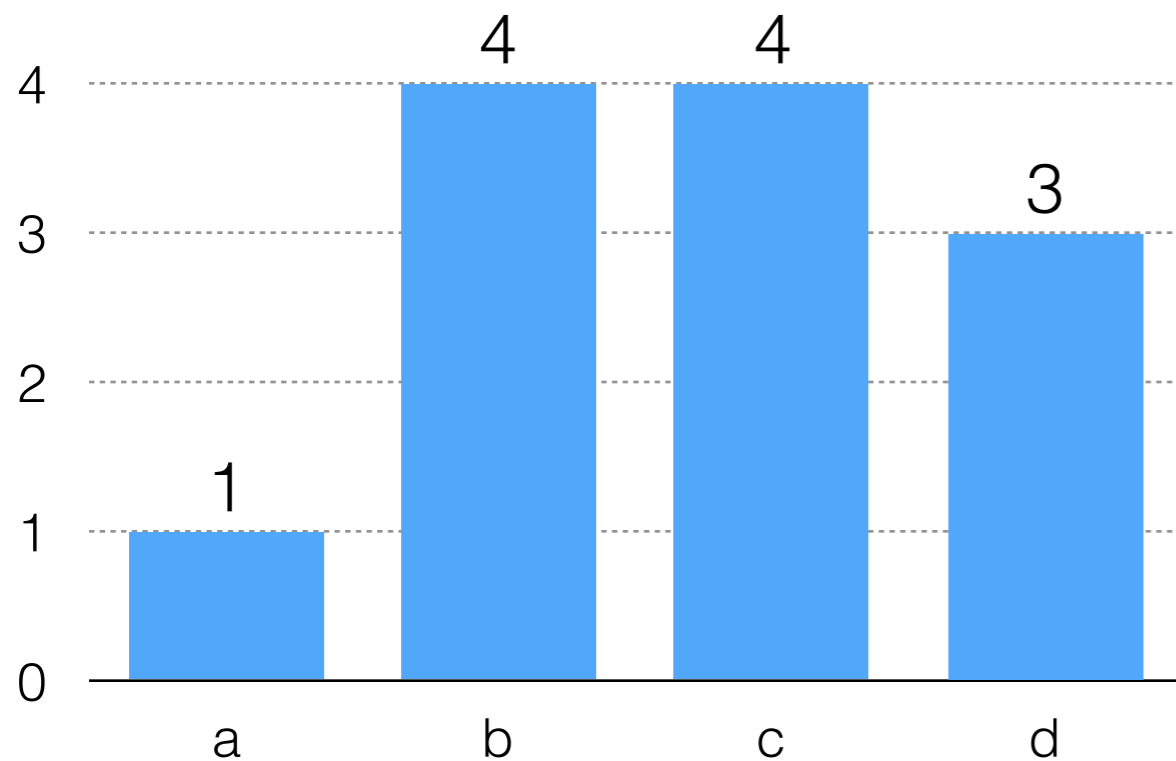


c	d	b	d
b	a	d	c
b	c	b	d

Are the answers to my clicker questions random?



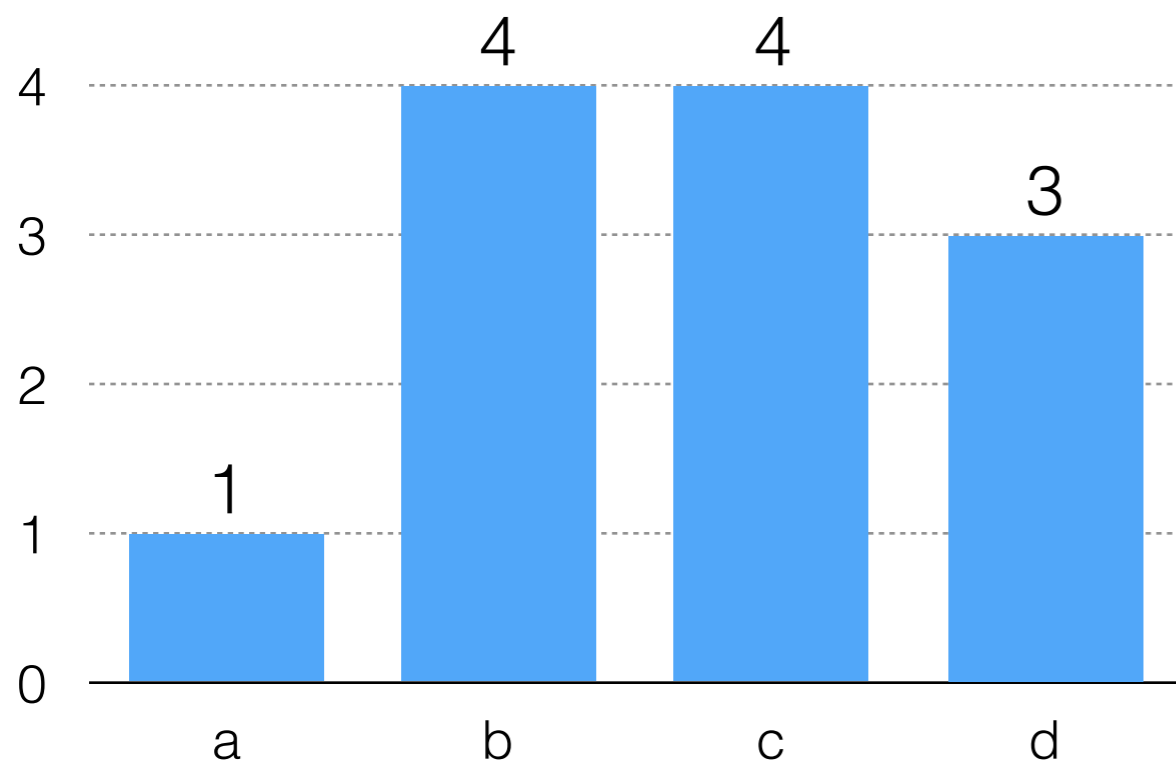
Are the answers to my clicker questions random?



X_i = count of answer i

$$p(a) = p(b) = p(c) = p(d) = 0.25$$

Are the answers to my clicker questions random?

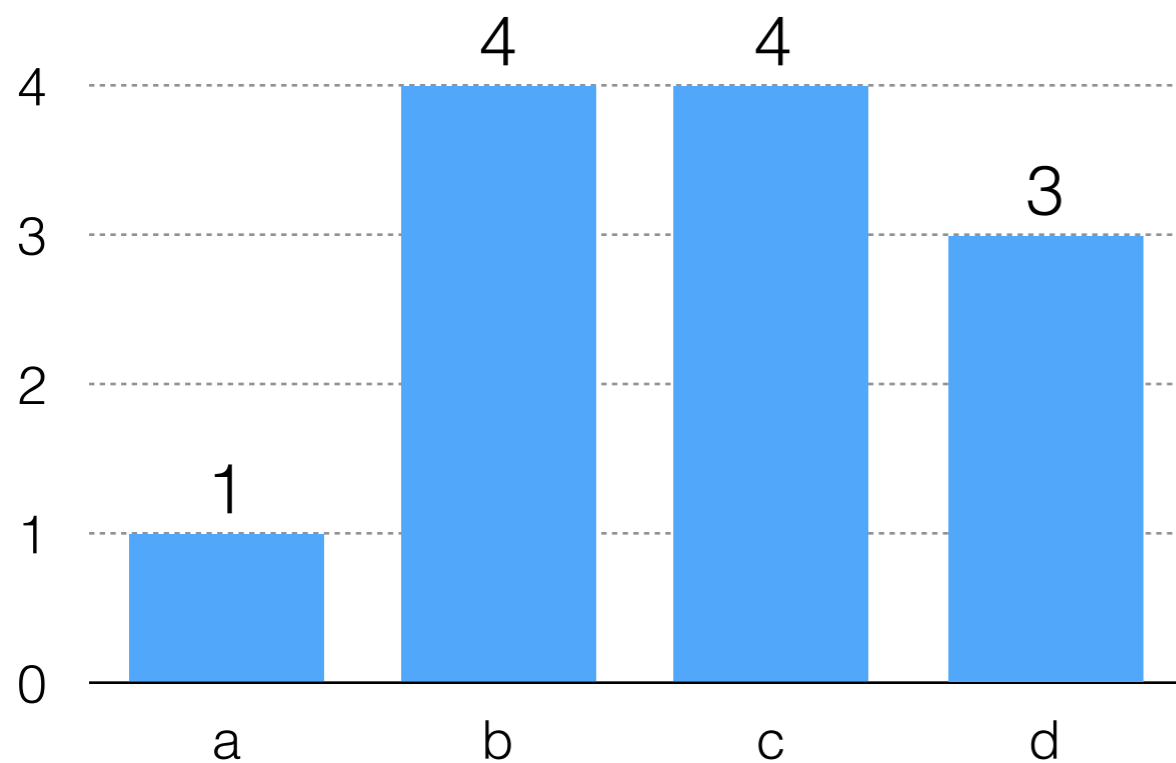


Is this distribution significantly different than what we would expect by chance, assuming that in fact all answers are equally likely?

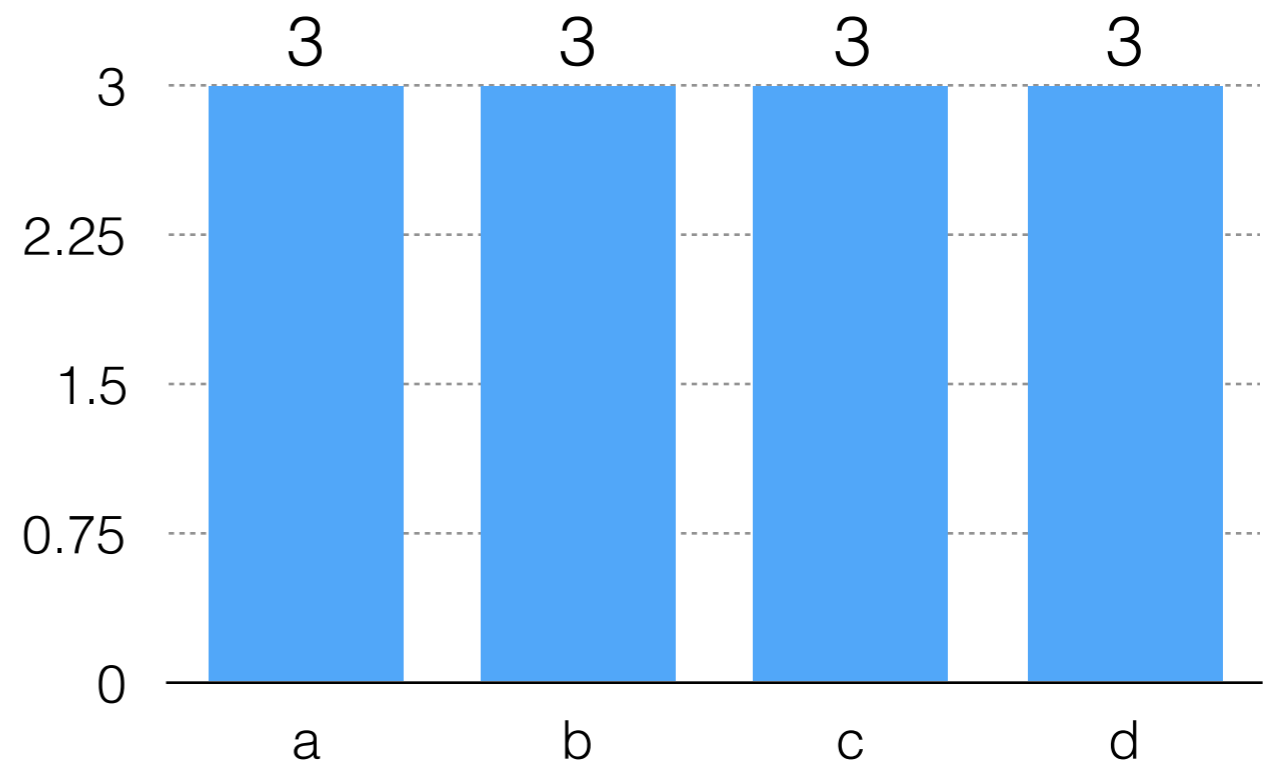
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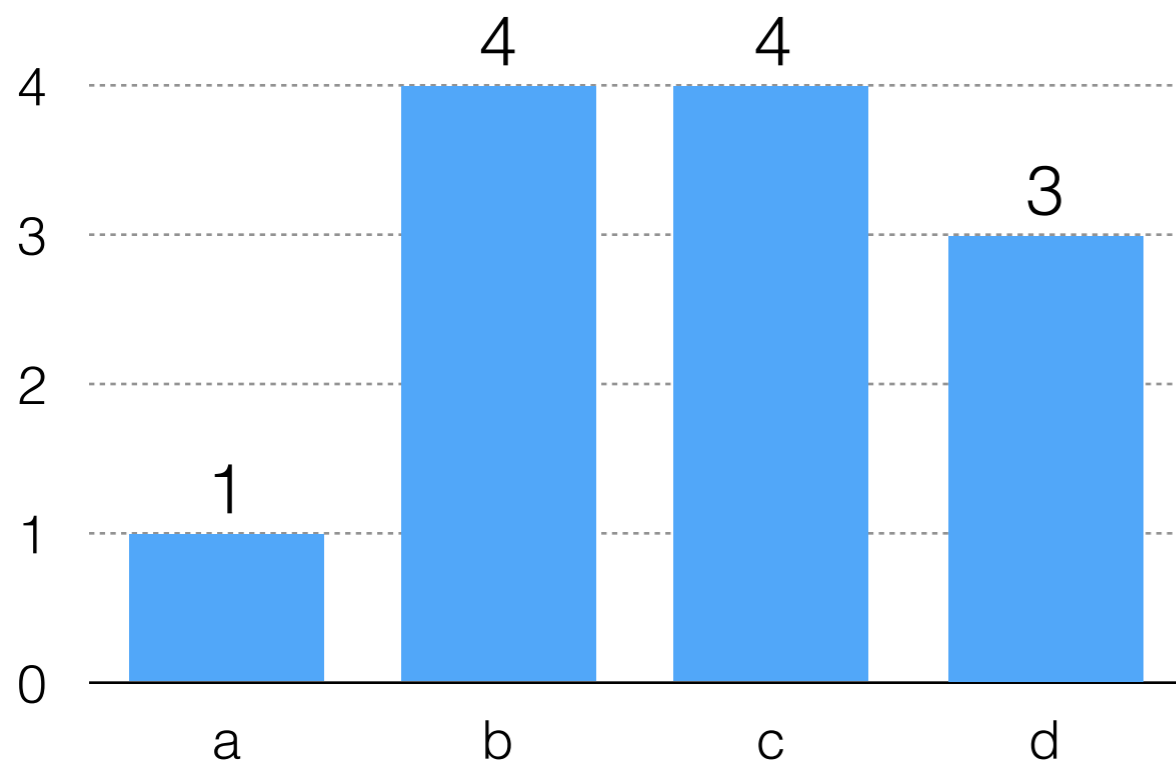


Observed

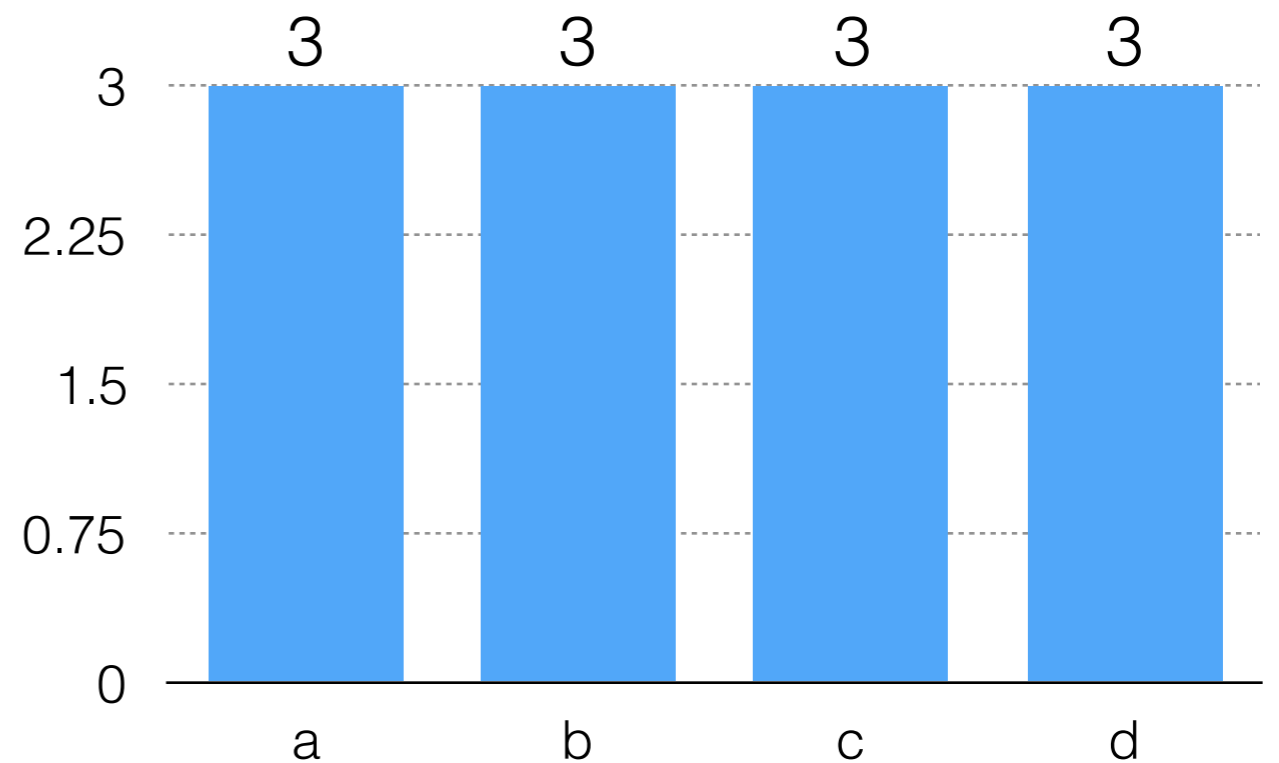


Expected

Are the answers to my clicker questions random?



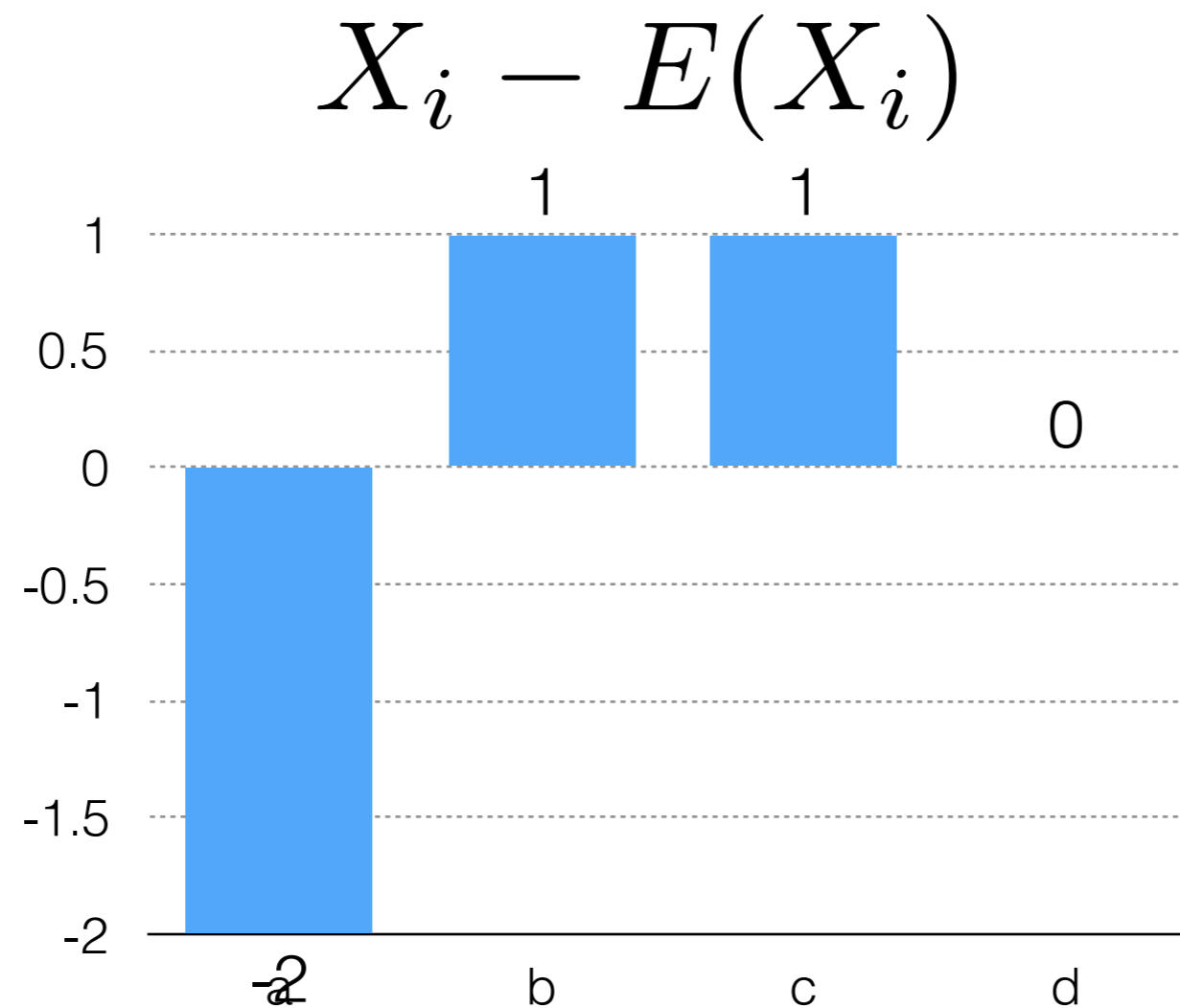
Observed



Expected

Want to model the difference between these

Are the answers to my clicker questions random?



Clicker Question!

Clicker Question!

Should I use the total difference between observed and expected as my summary statistic? I.e.

$$\sum_i (X_i - E(X_i))$$

- a) **Yes! That sounds good.**
- b) **No! I have qualms...**

Clicker Question!

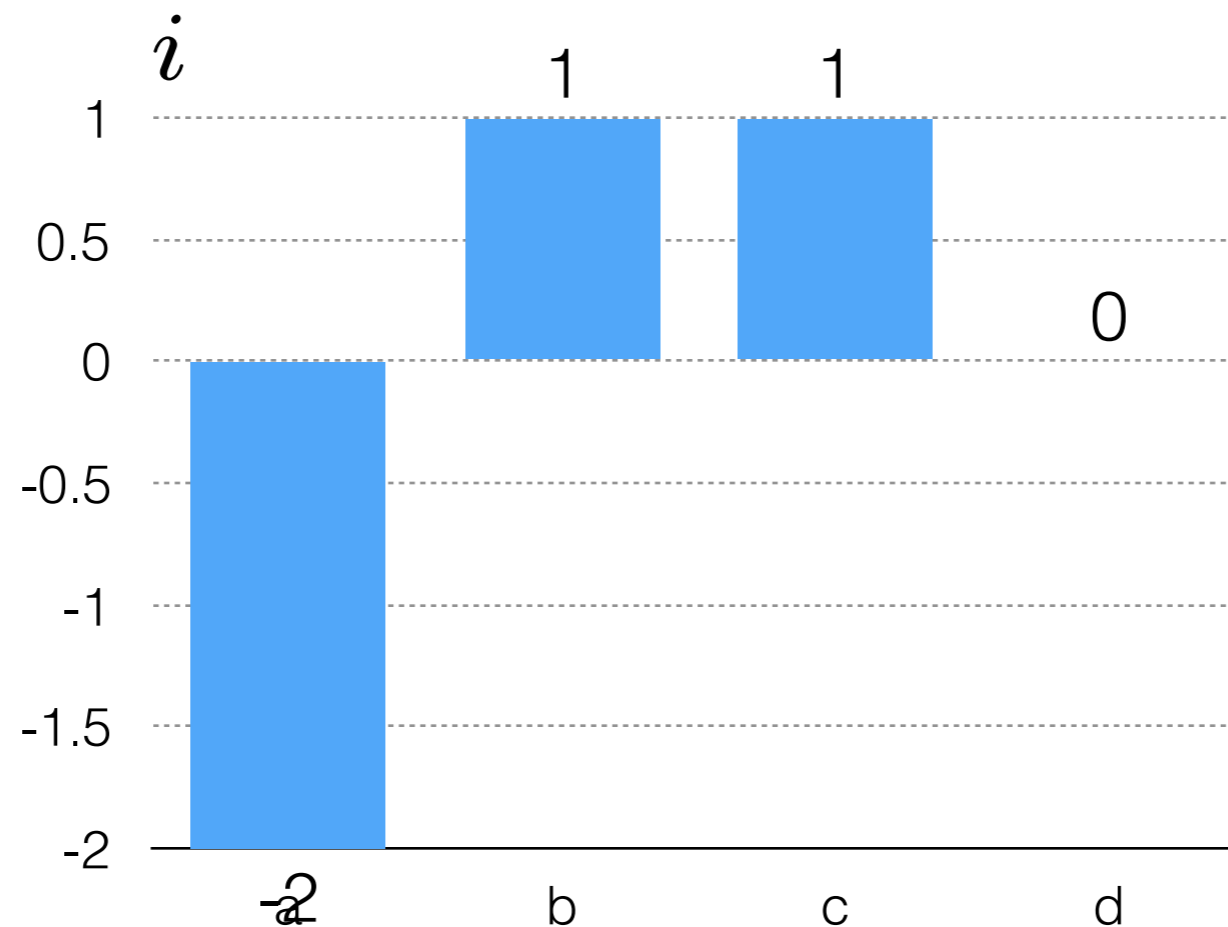
Should I use the total difference between observed and expected as my summary statistic? I.e.

$$\sum_i (X_i - E(X_i))$$

- a) Yes! That sounds good.
- b) No! I have qualms...

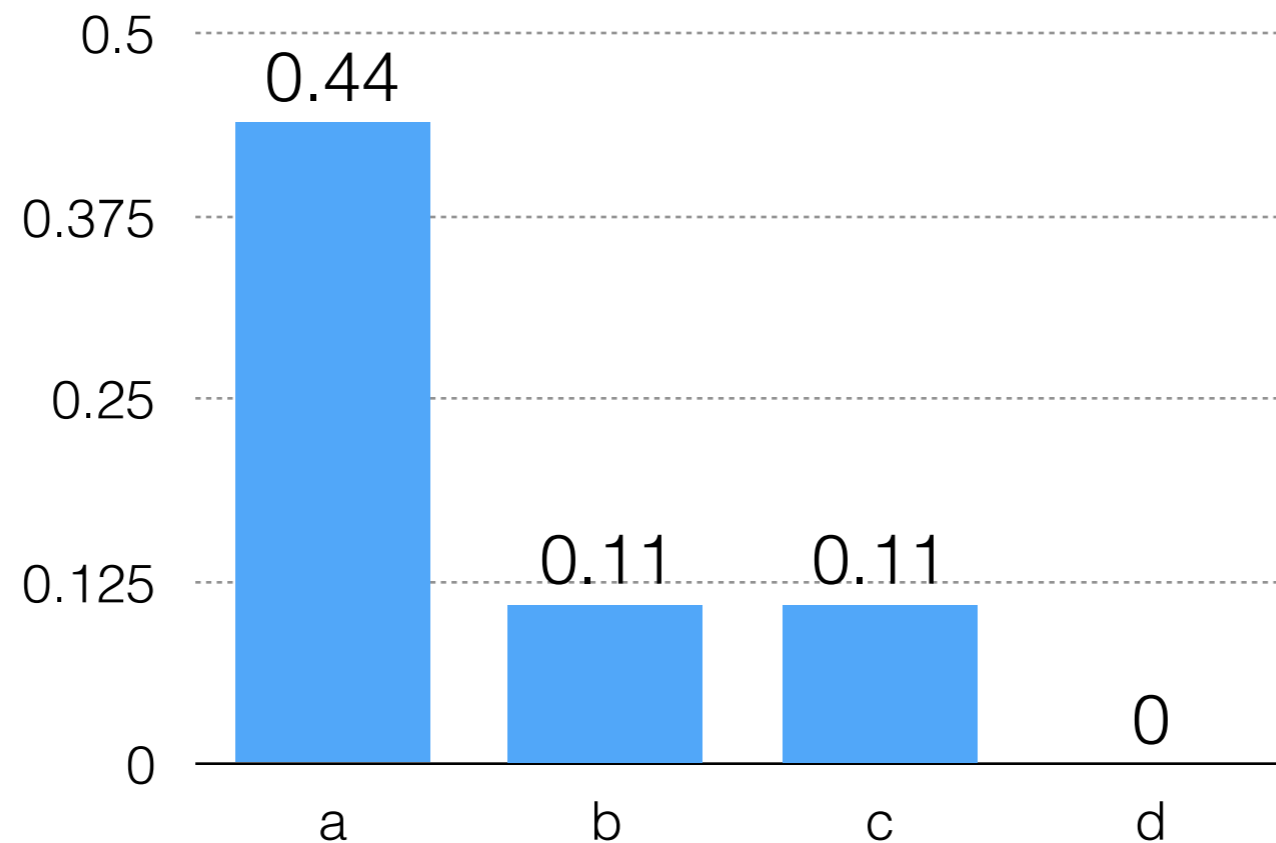
Are the answers to my clicker questions random?

$$\sum (X_i - E(X_i)) = 0$$



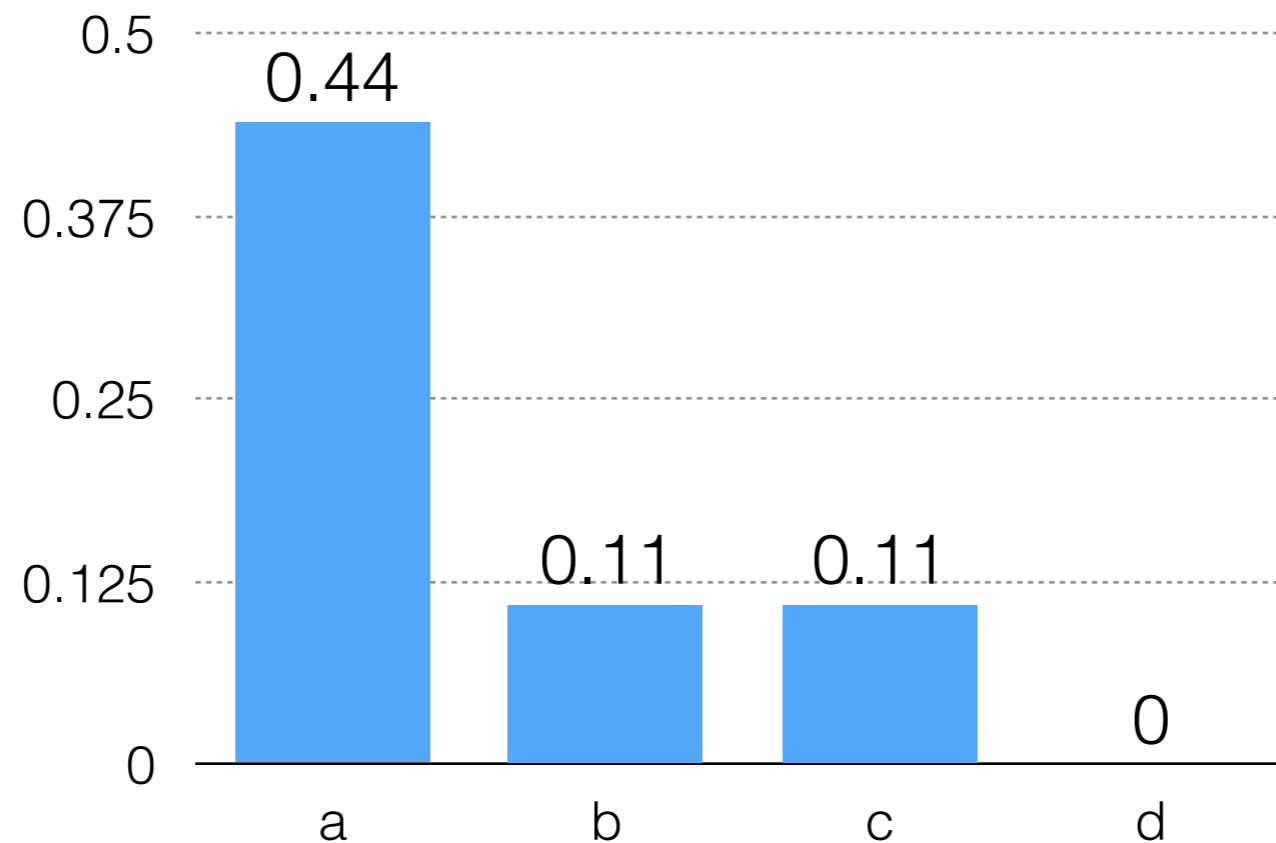
Are the answers to my clicker questions random?

$$\sum_i (X_i - E(X_i))^2$$



Are the answers to my clicker questions random?

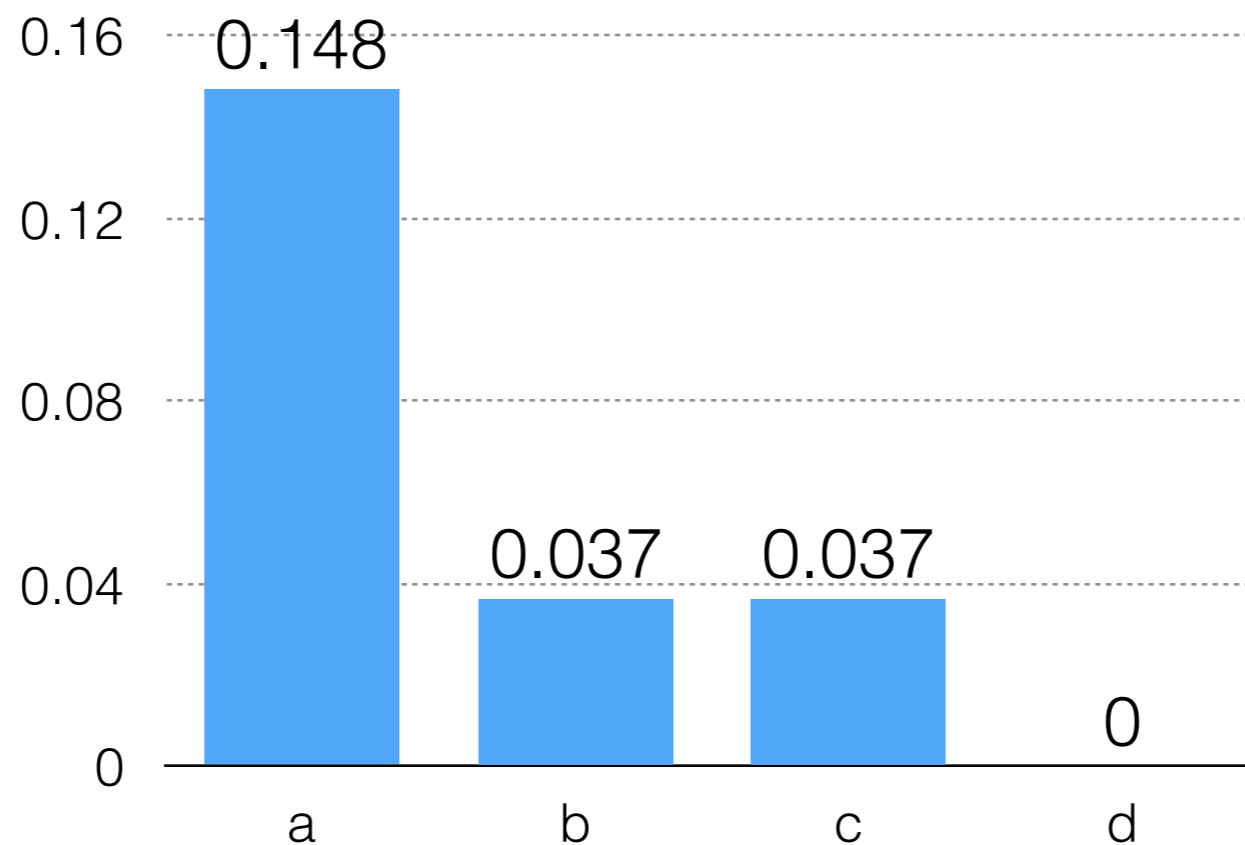
$$\sum_i (X_i - E(X_i))^2$$



Thoughts?

Are the answers to my clicker questions random?

$$\sum_i \frac{(X_i - E(X_i))^2}{E(X_i)}$$

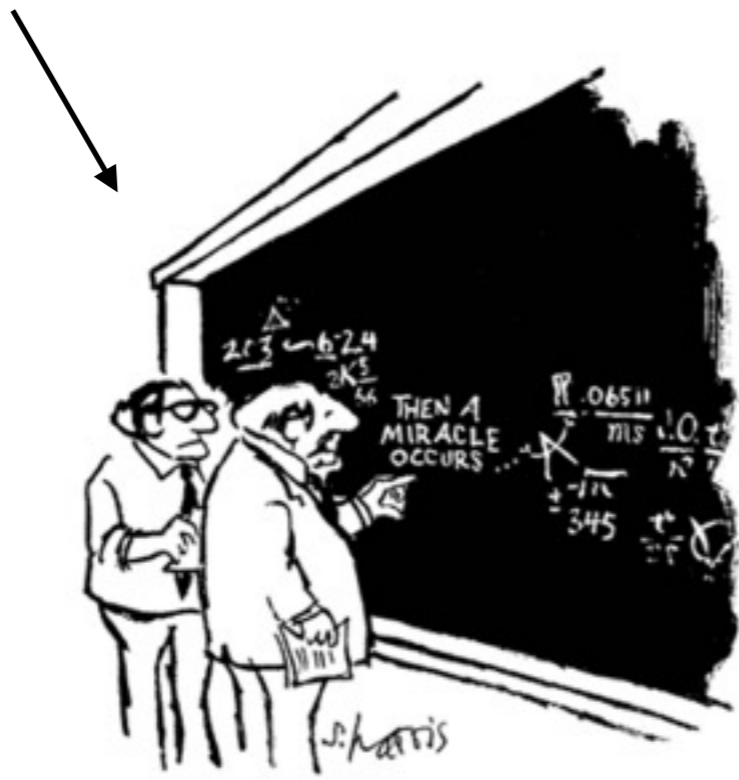


Are the answers to my
clicker questions random?

$$\sum_i \frac{(X_i - E(X_i))^2}{E(X_i)}$$

Are the answers to my clicker questions random?

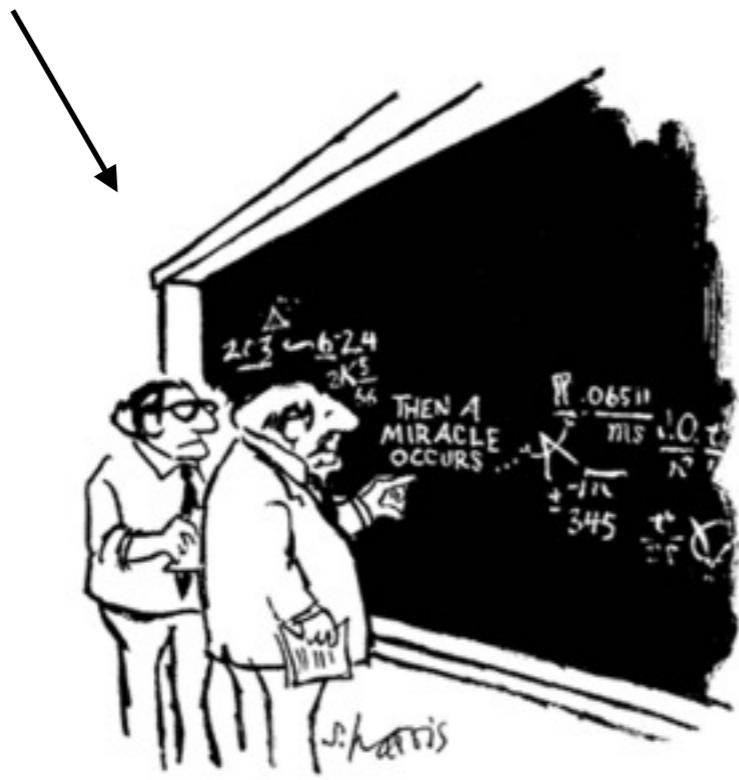
$$\sum_i \frac{(X_i - E(X_i))^2}{E(X_i)}$$



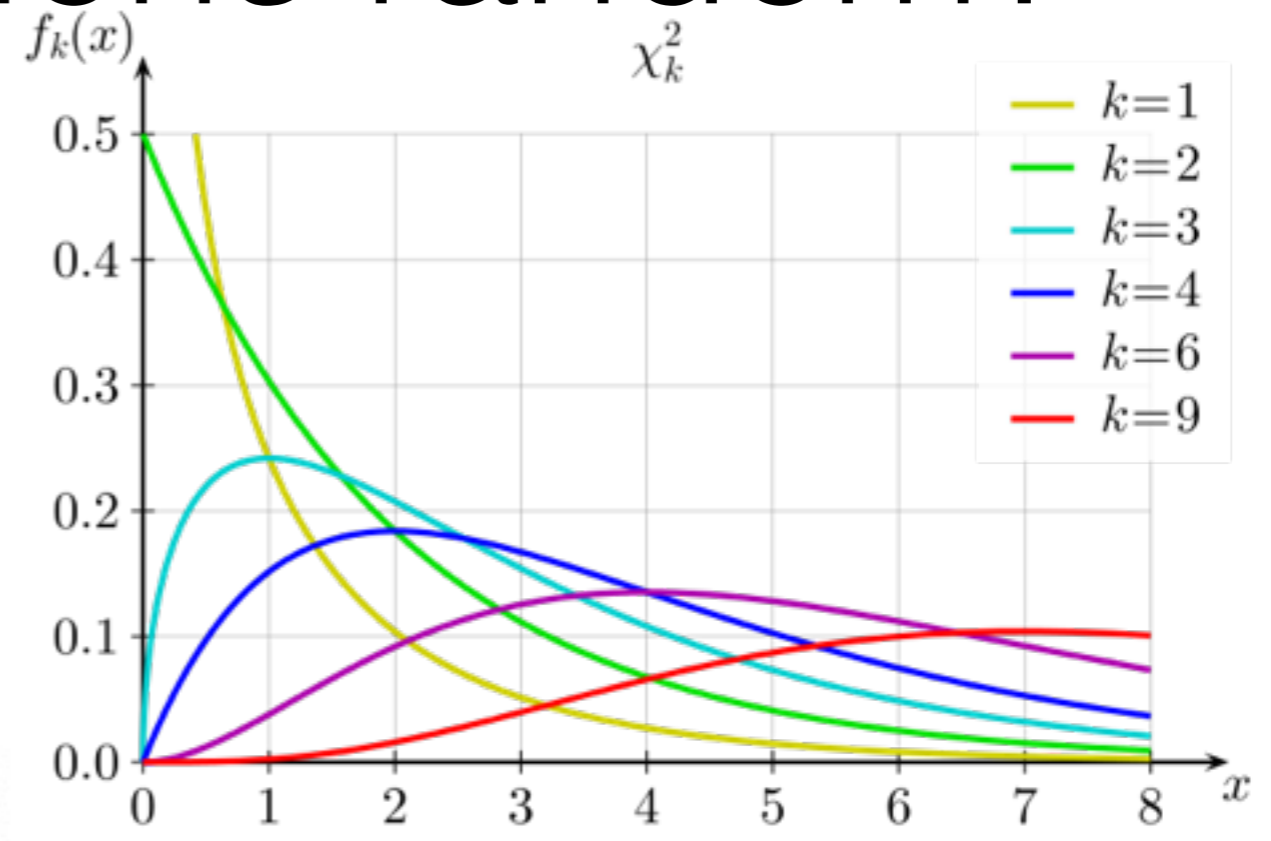
"I think you should be more explicit here in step two."

Are the answers to my clicker questions random?

$$\sum_i \frac{(X_i - E(X_i))^2}{E(X_i)}$$

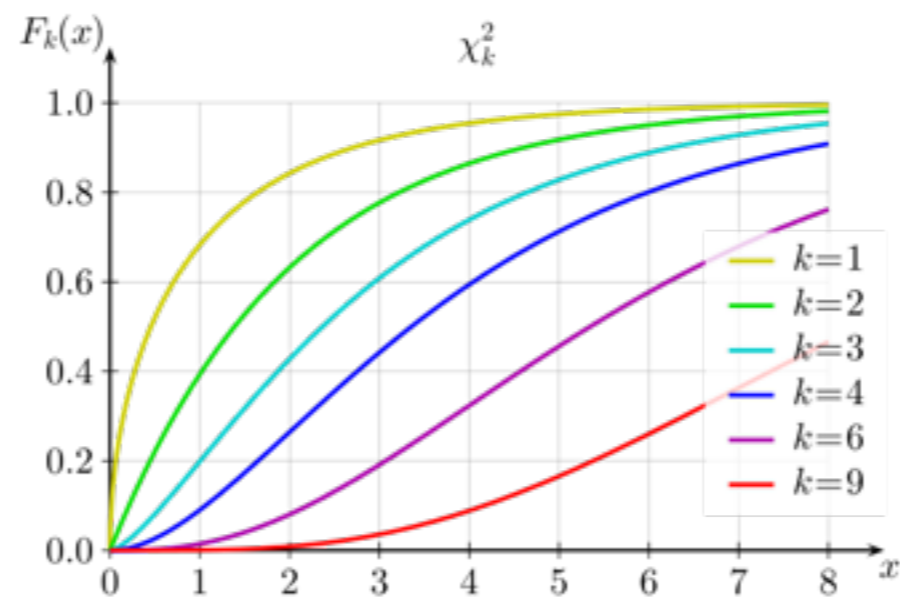


"I think you should be more explicit here in step two."



Chi Squared Test

$$\sum_i \frac{(X_i - E(X_i))^2}{E(X_i)}$$

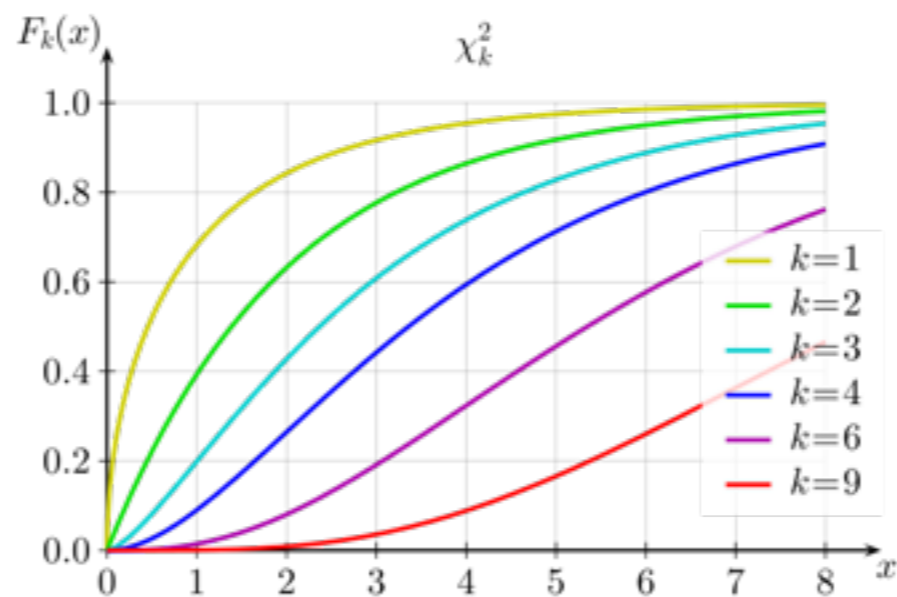
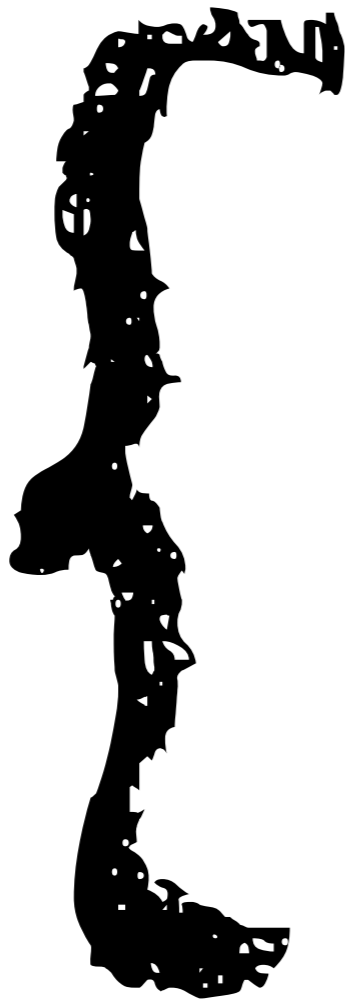


$$\frac{1}{\Gamma(k/2)} \gamma\left(\frac{k}{2}, \frac{x}{2}\right)$$

Chi Squared Test

$$\sum_i \frac{(X_i - E(X_i))^2}{E(X_i)}$$

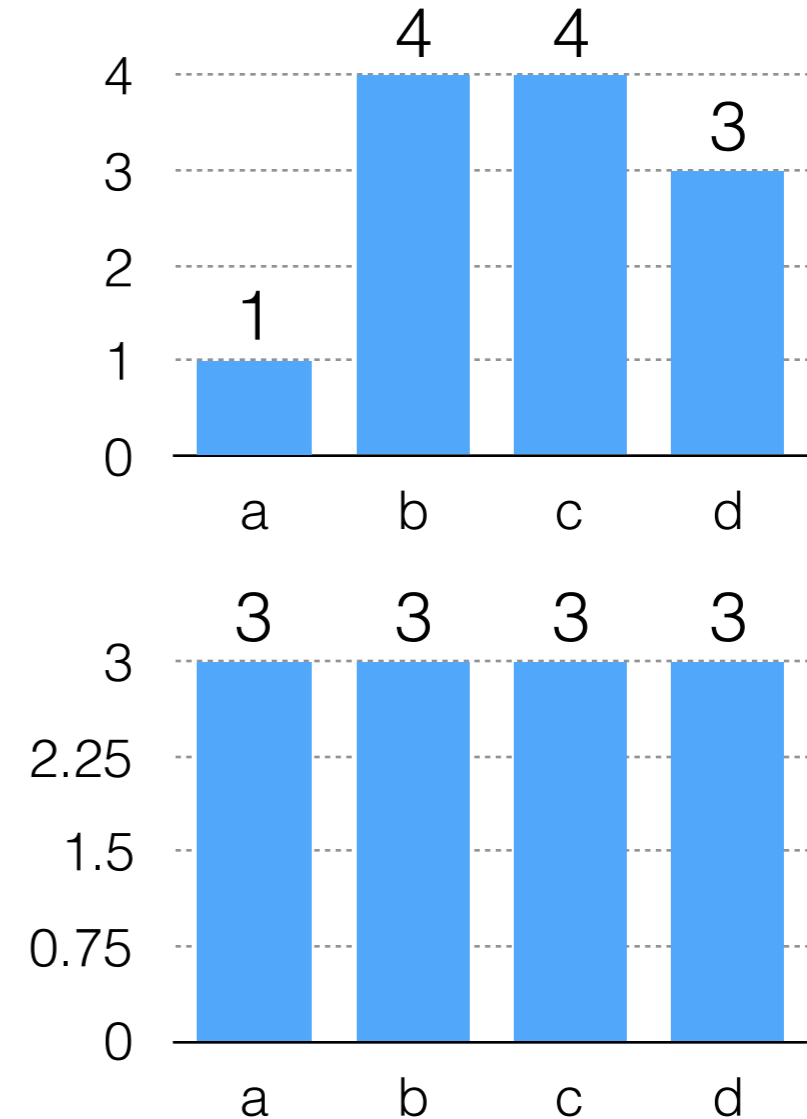
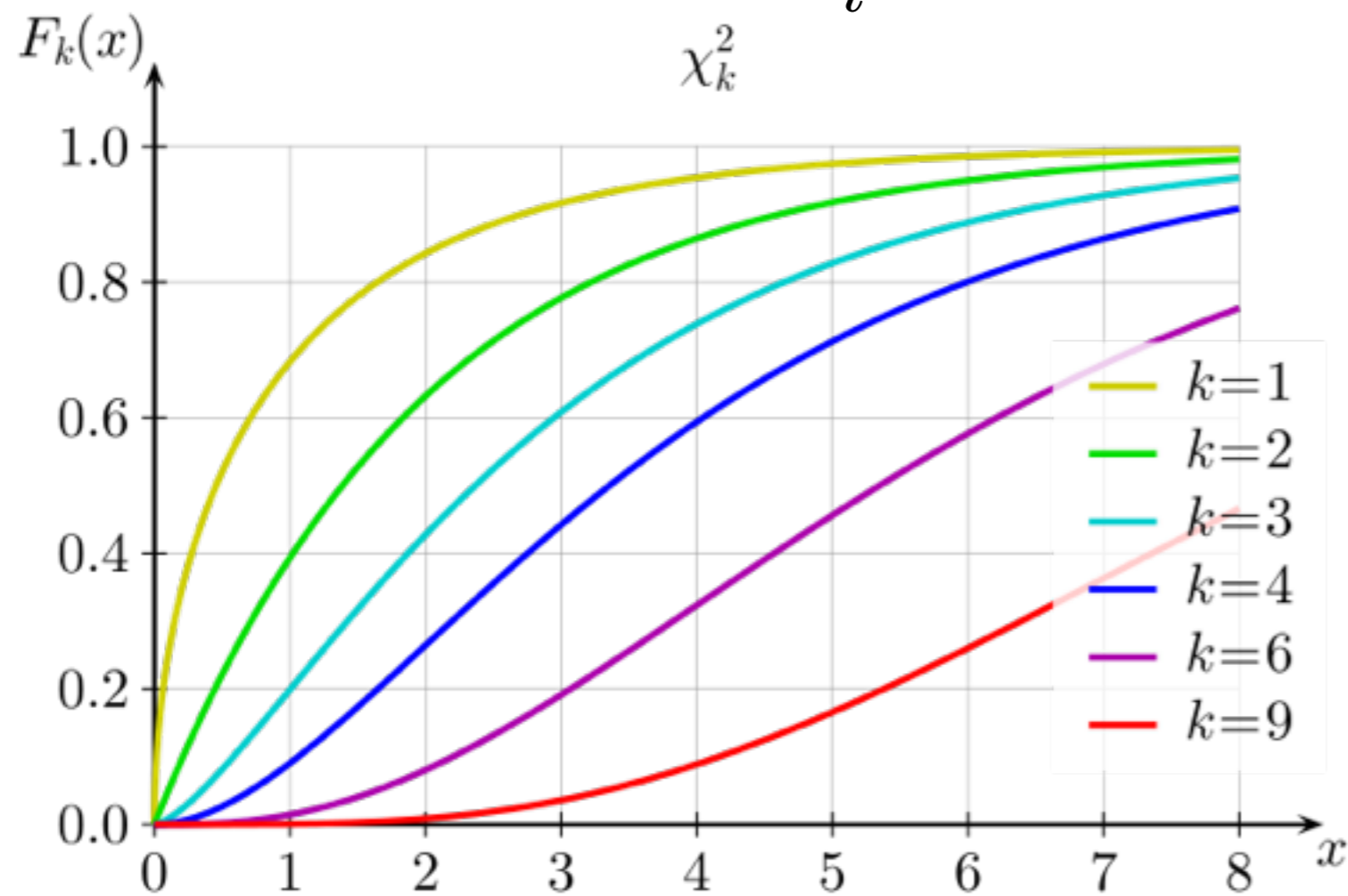
cdf
that we
can
compute
explicitly



$$\frac{1}{\Gamma(k/2)} \gamma\left(\frac{k}{2}, \frac{x}{2}\right)$$

Chi Squared Test

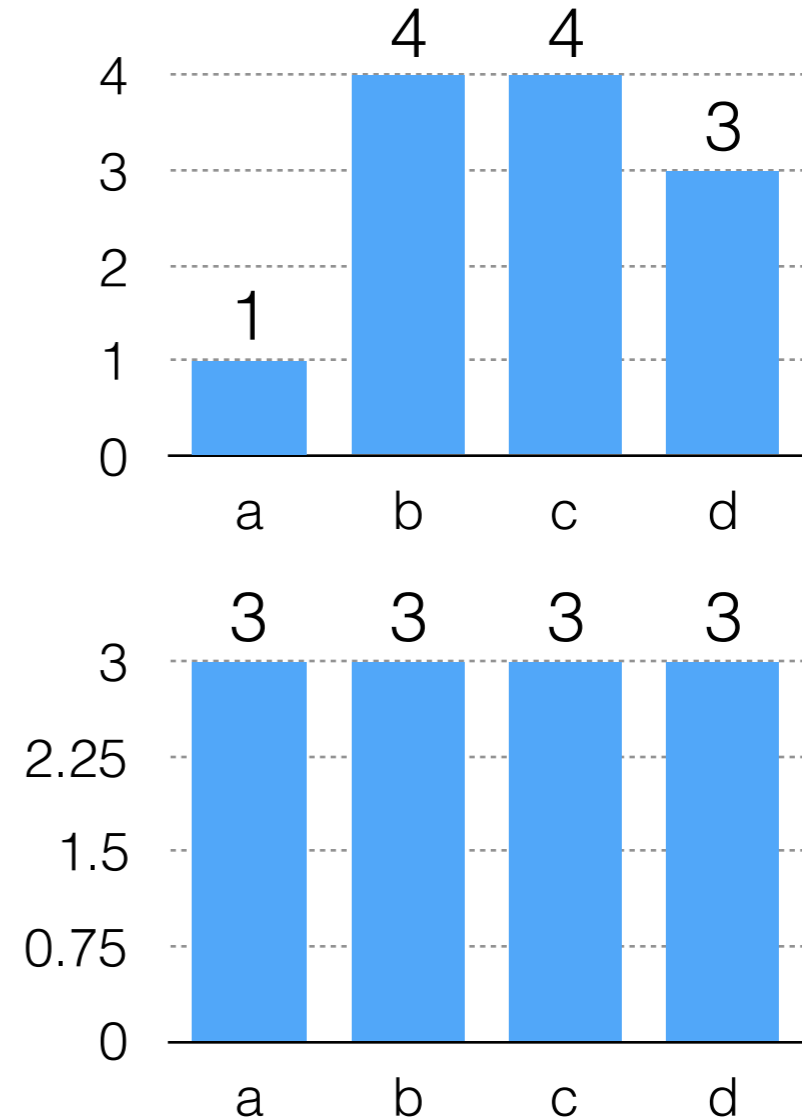
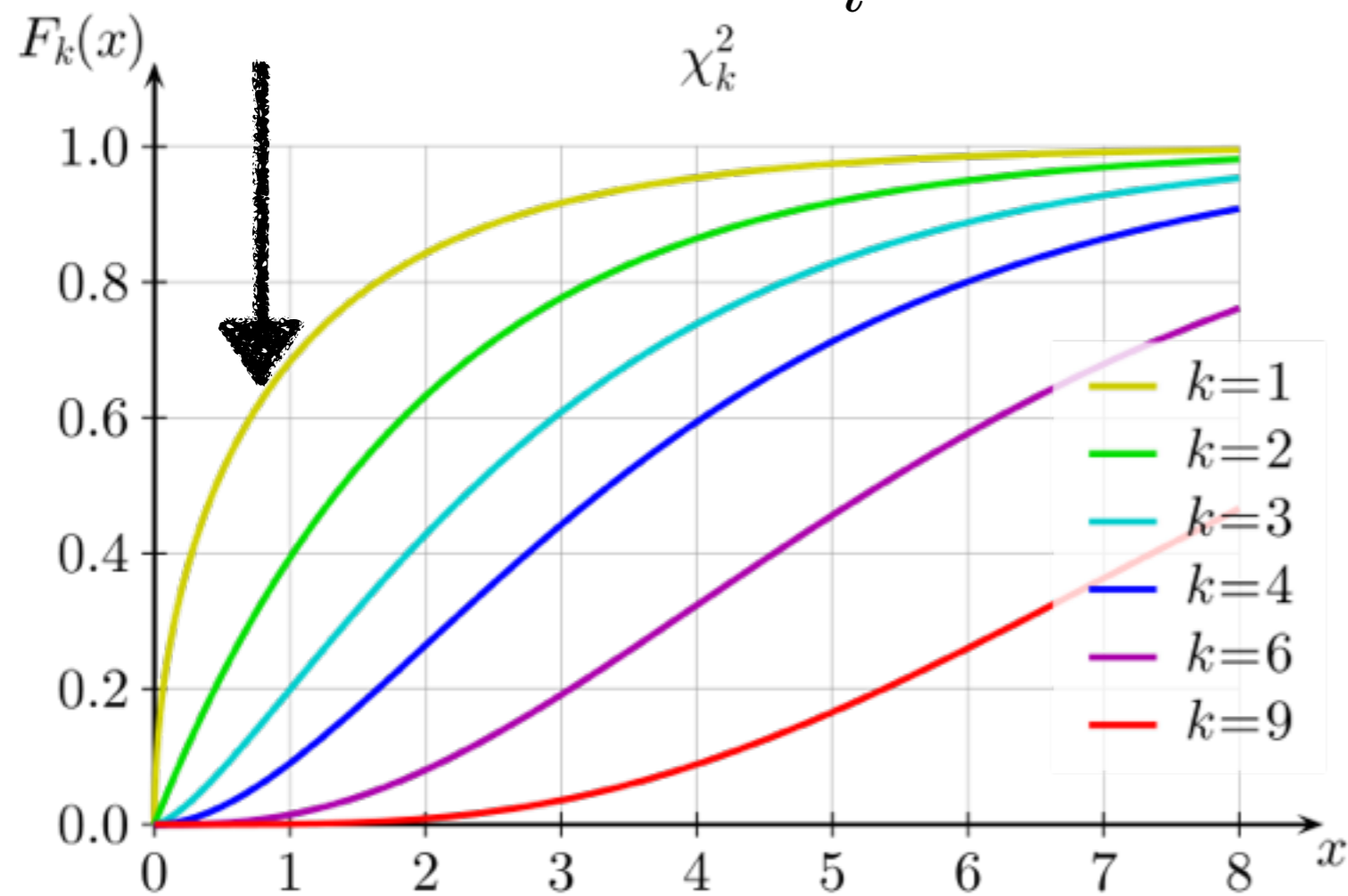
$$\sum_i \frac{(X_i - E(X_i))^2}{E(X_i)}$$



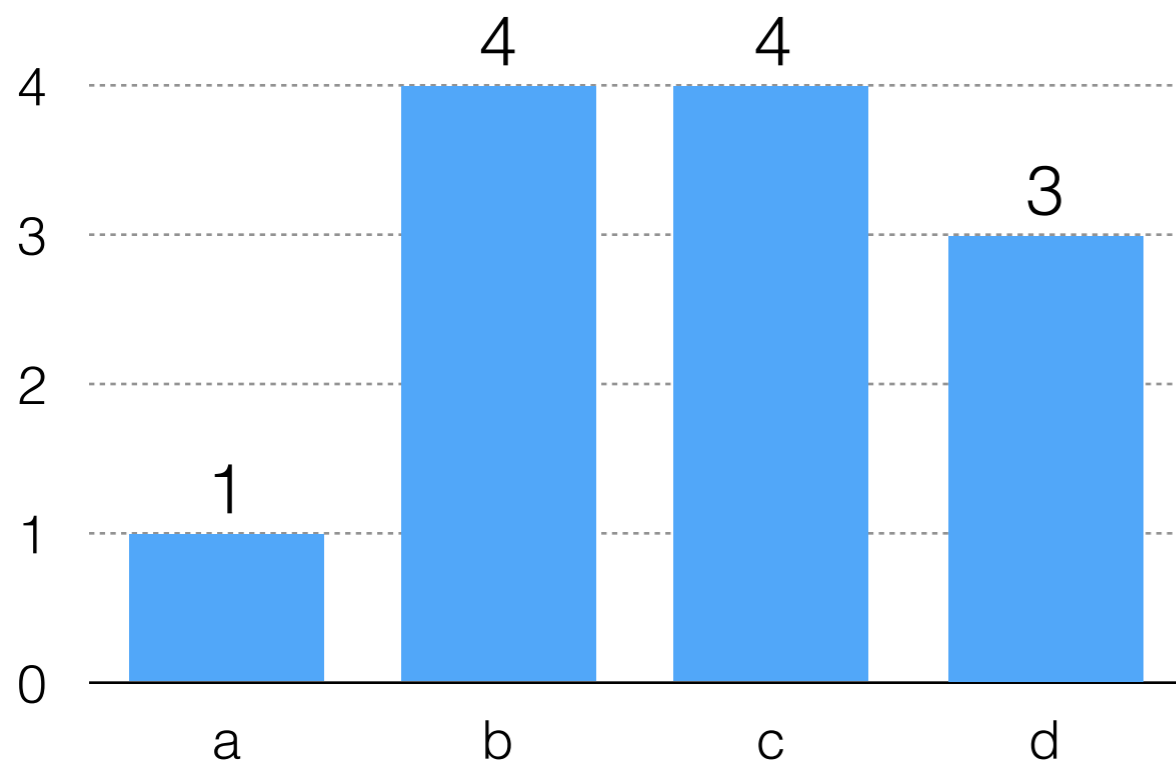
Chi Squared Test

not really remarkable

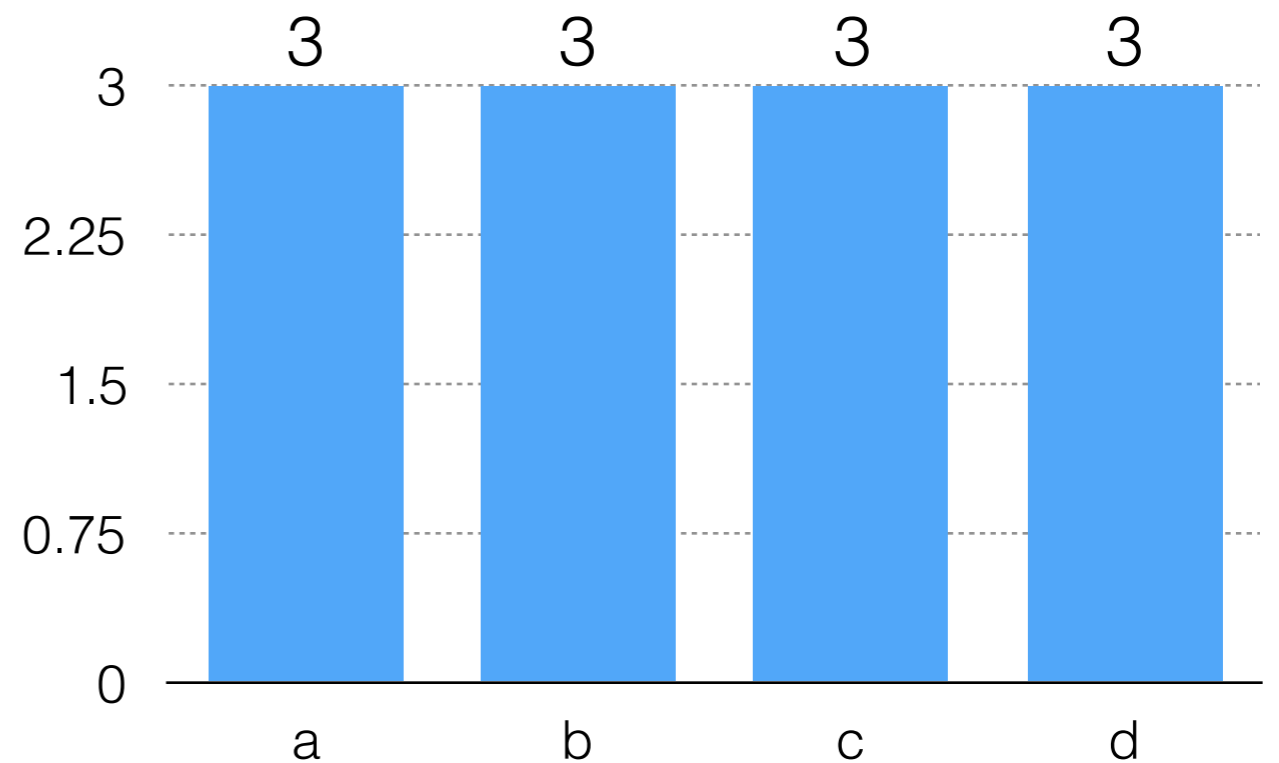
$$\sum_i \frac{(X_i - E(X_i))^2}{E(X_i)}$$



Chi Squared Test



Observed



Expected

Want to model the difference between these

Chi Squared Test

Overall goal: Understand how cities differ in terms of professions.

Null Hypothesis: No difference between Providence and Boston

	Art	Tech	Medicine
PVD	35	33	32
Boston	25	30	45

Chi Squared Test

Overall goal: Understand how cities differ in terms of professions.

Null Hypothesis: No difference between Providence and Boston

<i>"expected"</i>	Art	Tech	Medicine
PVD	35	33	32
Boston	25	30	45
<i>"observed"</i>			

Clicker Question!

Clicker Question!

Compute the test statistic

$$\sum_i \frac{(X_i - E(X_i))^2}{E(X_i)}$$

Null Hypothesis: No difference between Providence and Boston

"expected"

	Art	Tech	Medicine
PVD	35	33	32

Boston

25 30 45

"observed"

- a) 6.5
- b) 7.3
- c) 8.4

Clicker Question!

$$\sum_i \frac{(X_i - E(X_i))^2}{E(X_i)}$$

Compute the test statistic

Null Hypothesis: No difference between Providence and Boston

"expected"

Art

Tech

Medicine

PVD

35

33

32

Boston

25

30

45

"observed"

a) 6.5

b) 7.3

c) 8.4

Clicker Question!

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Compute the test statistic

chisquare

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"expected"

	Art	Tech	Medicine
PVD	35	33	32

Boston

25	30	45
----	----	----

"observed"

a) 6.5

b) 7.3

c) 8.4

okie done now